

Approximation and Estimation

Rounding, significant figures, approximation errors, estimation strategies and real-world applications.



For students	For teachers	Practice load
Revise the full chapter with clear notes, worked examples, practice questions and answers.	Use as a full lesson, worksheet pack or revision handout.	Expanded question bank designed for about 1.5 hours of student work, depending on class pace.

Built from the Chapter 3 topic flow: approximation versus estimation, rounding to decimal places and significant figures, approximation errors, estimation in real-world contexts, value-for-money comparisons and estimating larger quantities from smaller quantities.

Chapter 3 Learning Map

Approximation and estimation are used when exact numbers are unnecessary, unavailable, or too slow to obtain. This chapter also trains students to decide whether an answer is reasonable.

Section	What students should master	Typical question types
3.1 Rounding and significant figures	Round whole numbers and decimals to a required place value or number of significant figures.	Nearest 10, 100, 1000; decimal places; significant figures; largest/smallest possible actual values.
3.2 Approximation errors	Understand that rounding too early can cause follow-through errors in later working.	Intermediate working; degree of accuracy; approximate values in real-world contexts.
3.3 Estimation and estimation errors	Use mental methods and sensible approximations to estimate answers and check whether calculator answers are reasonable.	Currency conversion, value for money, area estimates, larger quantity estimates.

90-minute practice plan

Time	Activity	Notes
10 min	Concept recap	Use pages 3–9 to revise key ideas.
25 min	Practice Set A and B	Rounding, decimal places and significant figures.
25 min	Practice Set C and D	Approximation errors and estimation.
20 min	Review Exercise	Mixed questions under test-like conditions.
10 min	Check answers	Use selected explanations for common mistakes.

1. Actual, Approximate and Estimated Values

Students must distinguish between exact values, approximate values and estimated values. This distinction appears throughout the chapter.

Actual value

The exact value, when it is known or measured accurately.
Example: a measured distance of 18.6 km.

Approximate value

A value obtained by rounding an actual or measured value.
Example: $18.6 \text{ km} \approx 19 \text{ km}$.

Estimated value

A sensible guess or mental calculation made when the exact value is not needed or not known.
Example: about 10 km.

Key distinction

Approximation normally starts with a known value and rounds it to a convenient degree of accuracy.

Estimation normally starts with incomplete information and gives a sensible value based on mental reasoning.

Both are useful, but they are not the same.

Situation	Better description	Reason
A website gives the distance as 18.6 km, then someone says it is about 19 km.	Approximation	The value 18.6 km is rounded to a nearby value.
A student guesses that a journey is about 10 km before checking the exact distance.	Estimation	The student is making a sensible guess.
A scientist records a measured mass as 42.13 g.	Actual / measured value	It is the recorded measured value before rounding.

2. Rounding Whole Numbers and Decimals

Rounding makes a number easier to use. The required place value tells us where the final rounded digit should be.

If the next digit is...	What to do	Example
0, 1, 2, 3 or 4	Keep the selected digit unchanged.	273.1 to nearest 10 = 270
5, 6, 7, 8 or 9	Increase the selected digit by 1.	275 to nearest 10 = 280

Worked example 1: Round 698 352

To nearest 10: look at the ones digit 2 → 698 350.

To nearest 100: look at the tens digit 5 → 698 400.

To nearest 1000: look at the hundreds digit 3 → 698 000.

Worked example 2: Correct 45.7395

1 decimal place: look at hundredths digit 3 → 45.7.

Nearest whole number: look at tenths digit 7 → 46.

Nearest 0.01: look at thousandths digit 9 → 45.74.

Nearest thousandth: look at the next digit 5 → 45.740.

3. Significant Figures

Significant figures show the degree of accuracy of a number. Students need to know which digits count as significant.

Rule	Meaning	Example
Rule 1	All non-zero digits are significant.	783 has 3 significant figures.
Rule 2	Zeros between non-zero digits are significant.	700.406 has 6 significant figures.
Rule 3	Zeros before non-zero digits in decimals are not significant.	0.003 019 has 4 significant figures.
Rule 4	Zeros after non-zero digits in decimals are significant.	13.00 has 4 significant figures.
Rule 5	Zeros at the end of a whole number may or may not be significant, depending on how the number is rounded.	7500 can have different possible numbers of significant figures.

Worked example 3: State the number of significant figures

39 018 → 5 significant figures.

0.028 030 → 4 significant figures: 2, 8, 0 and 3.

700.406 00 → 8 significant figures.

2900 to nearest 100 → 2 significant figures.

4. Rounding to Significant Figures

To round to significant figures, count from the first non-zero digit, not from the decimal point.

Step	Action
1	Find the first significant digit. It is the first non-zero digit.
2	Count to the required number of significant figures.
3	Look at the next digit. If it is 5 or more, round up.
4	Use zeros as place holders where necessary.

Worked example 4

8982 to 2 s.f. $\rightarrow$ 9000.

0.0060195 to 4 s.f. $\rightarrow$ 0.006020.

0.9999 to 3 s.f. $\rightarrow$ 1.00.

503.98 to 4 s.f. $\rightarrow$ 504.0.

Common mistake	Correction
Writing 1 instead of 1.00 for 0.9999 to 3 s.f.	1 has 1 significant figure. 1.00 has 3 significant figures.
Dropping zeros that show accuracy	Zeros after non-zero digits in decimals are significant.
Counting leading zeros	Zeros before the first non-zero digit do not count.

5. Approximation Errors

Approximation errors may happen when non-exact intermediate values are rounded too early. This can affect the final answer.

Situation	Possible problem	Better habit
Rounding intermediate working too early	The final answer may drift away from the accurate value.	Keep at least 4 or 5 significant figures in intermediate steps.
Using rounded percentages that should add to 100%	The total may become 99.9% or 100.1%.	Use exact values for calculation when possible.
Using 11.4 instead of $\sqrt{131}$ too early	Further multiplication may create a follow-through error.	Store calculator values or use more significant figures.

Worked example 5: Square perimeter

Area of a square is 131 cm^2 .

Side length = $\sqrt{131} \text{ cm} \approx 11.445\ldots \text{ cm}$.

Perimeter = $4\sqrt{131} \text{ cm} \approx 45.8 \text{ cm}$ to 3 s.f.

Avoid using 11.4×4 if the final answer needs 3 s.f.

6. Degree of Accuracy and Bounds

When a value is given to a stated degree of accuracy, the actual value lies within a range.

Given value	Possible actual values	Example
450 000 to nearest 10 000	$445\,000 \leq \text{actual value} < 455\,000$	The smallest possible integer is 445 000; the largest is 454 999.
5.7 cm to nearest 0.1 cm	$5.65 \text{ cm} \leq \text{actual length} < 5.75 \text{ cm}$	5.65 rounds to 5.7, while 5.75 rounds to 5.8.
0.120 cm to 3 s.f.	Half-unit range around 0.120 at the last significant digit	Examples include values such as 0.1196 cm and 0.1204 cm.

Worked example 6

A population is approximately 500 000 to nearest 10 000.

Smallest possible whole-number population: 495 000.

Largest possible whole-number population: 504 999.

7. Estimation in Real-World Contexts

Estimation is used to check whether an answer is reasonable or to make quick decisions without exact calculation.

Context	Useful strategy	Example
Computation	Round each number to 1 or 2 significant figures.	$31.5 + 9.87 - 2.1 \approx 32 + 10 - 2 = 40$.
Currency conversion	Use a convenient conversion rate.	225 baht at about 25 baht per dollar $\approx$ \$9.
Value for money	Compare price per unit.	Find approximate cost per 100 g or per litre.
Larger quantities	Estimate a small sample, then scale up.	Count coins in a small box section, then multiply.
Area	Compare shaded and unshaded parts roughly.	Use visual partitioning to estimate ratios.

Worked example 7: Estimation without a calculator

$$31.5 + 9.87 - 2.1 \approx 32 + 10 - 2 = 40.$$

If a calculator gives 392.7, the answer is clearly not reasonable.

Practice Set A: Rounding whole numbers and decimals (24 questions)

1. Round 49 276 to the nearest 10.	2. Round 49 276 to the nearest 100.
3. Round 49 276 to the nearest 1000.	4. Round 49 276 to the nearest 10 000.
5. Round 698 352 to the nearest 10.	6. Round 698 352 to the nearest 100.
7. Round 698 352 to the nearest 1000.	8. Round 698 352 to the nearest 10 000.
9. Correct 78.4695 to 1 decimal place.	10. Correct 78.4695 to the nearest whole number.
11. Correct 78.4695 to the nearest 0.01.	12. Correct 78.4695 to the nearest 0.001.
13. Correct 7.697 146 to the nearest whole number.	14. Correct 7.697 146 to 2 decimal places.
15. Correct 7.697 146 to the nearest tenth.	16. Correct 7.697 146 to the nearest 0.000 01.
17. Round \$10.982 to the nearest ten cents.	18. Round 4.918 m to the nearest 0.1 m.
19. Round 9.71 cm to the nearest cm.	20. Round 6.489 kg to the nearest 0.1 kg.
21. Round 6.489 kg to the nearest 0.01 kg.	22. Round 0.04976 to 2 decimal places.
23. Round 0.04976 to 3 decimal places.	24. Round 999.95 to the nearest whole number.

Practice Set B: Significant figures (24 questions)

1. State the number of significant figures in 39 018.	2. State the number of significant figures in 0.028 030.
3. State the number of significant figures in 700.406 00.	4. State the number of significant figures in 2900, correct to nearest 10.
5. State the number of significant figures in 2900, correct to nearest 100.	6. State the number of significant figures in 13.00.
7. State the number of significant figures in 0.001 740.	8. State the number of significant figures in 80 070.
9. Round 728 to 2 significant figures.	10. Round 503.98 to 4 significant figures.
11. Round 0.003 0195 to 4 significant figures.	12. Round 6396 to 2 significant figures.
13. Round 6396 to 3 significant figures.	14. Round 9.9999 to 3 significant figures.
15. Round 8.004 to 3 significant figures.	16. Round 0.050 90 to 3 significant figures.
17. Round 21 507 to 2 significant figures.	18. Round 21 507 to 3 significant figures.
19. Round 0.000 987 65 to 2 significant figures.	20. Round 0.000 987 65 to 4 significant figures.
21. Round 5.995 to 3 significant figures.	22. Round 999 950 to 3 significant figures.
23. Round 999 950 to 4 significant figures.	24. Write 0.02 to 3 significant figures.

Practice Set C: Approximation errors and bounds (18 questions)

1. A population is 5 710 000 to nearest 10 000. Find the smallest possible integer value.	2. A population is 5 710 000 to nearest 10 000. Find the largest possible integer value.
3. A length is 7.4 cm to nearest 0.1 cm. State the range of possible values.	4. A mass is 2.35 kg to nearest 0.01 kg. State the range of possible values.
5. A distance is approximately 19 km to nearest km. Find the smallest possible distance.	6. A distance is approximately 19 km to nearest km. Find the largest possible distance less than 19.5 km.
7. A floor area is 225 000 m ² to 5 significant figures. Find the smallest possible integer value.	8. A floor area is 225 000 m ² to 5 significant figures. Find the largest possible integer value.
9. A diameter is 0.120 cm to 3 significant figures. Give three possible actual values.	10. A timing is recorded as 50.4 s to nearest 0.1 s. Give three possible actual timings.
11. The value 143 000 is correct to x significant figures. Write the possible values of x.	12. Explain why 5192.3 is not 519 when rounded to nearest 10.
13. Explain why 26.97 to 1 decimal place should be written as 27.0.	14. Explain why 0.0200 has 3 significant figures.
15. The side of a square is $\sqrt{105}$ cm. Estimate the perimeter to 3 s.f. without rounding the side too early.	16. Why should intermediate working usually keep more significant figures than the final answer?
17. A percentage table gives 62.3%, 17.3% and 20.3%. What total do these add to?	18. Explain how rounding each percentage can create a missing 0.1%.

Practice Set D: Estimation in real-world contexts (18 questions)

1. Estimate $31.5 + 9.87 - 2.1$ without a calculator.	2. Estimate 79.8×195 without a calculator.
3. Estimate $302 \div 29.7$ without a calculator.	4. Estimate 4.98×21.3 without a calculator.
5. A wallet costs 225 baht. If 25 baht is about S\$1, estimate the price in Singapore dollars.	6. A pair of earrings costs 25 000 rupiah. If 10 000 rupiah is about S\$1, estimate the price in Singapore dollars.
7. Option A: 200 g for \$6. Option B: 250 g for \$7.45. Which is better value?	8. Option A: 300 ml for \$4.80. Option B: 500 ml for \$7.80. Which is better value?
9. A bus can carry 30 passengers. Estimate how many buses are needed for 215 students and 5 teachers.	10. Each ticket costs \$4.90. Estimate the total cost for 38 students.
11. A car travels 274 km on 21 litres of petrol. Estimate km per litre.	12. A shopkeeper orders 32 skirts at \$19 each. Estimate the cost to nearest \$100.
13. A school event needs 1 bottle of water for 4 students. Estimate bottles needed for 238 students.	14. A tank is filled with 10-cent coins. A small sample suggests about 65 coins per layer and 18 layers. Estimate the value.
15. Estimate the shaded percentage if about 3 out of 8 equal parts are shaded.	16. Estimate the ratio of shaded to unshaded area if the shaded part is about three times the unshaded part.
17. Estimate $\sqrt{50}$ to 1 decimal place using nearby square numbers.	18. Explain why estimation can help detect calculator entry errors.

Review Exercise 3: Mixed practice (24 questions)

1. Round 6 479 952 to nearest 100.	2. Round 6 479 952 to nearest 10 000.
3. Correct 96.482 to 1 decimal place.	4. Correct 96.482 to the nearest whole number.
5. Round 0.006 020 to 2 significant figures.	6. Round 0.006 020 to 4 significant figures.
7. State the number of significant figures in 0.109 08.	8. State the number of significant figures in 25 000 km, correct to nearest km.
9. Correct 698.352 to 3 significant figures.	10. Correct 698.352 to 4 significant figures.
11. A value is 48.6 kg to nearest 0.1 kg. State the possible range.	12. A value is 9000 to nearest 1000. State the possible range of whole-number values.
13. Evaluate an estimate for $145.6 + 65.4$ without a calculator.	14. Estimate 14.5×65.4 without a calculator.
15. A tourist says the airport is about 10 km away, but a website gives 18.6 km. Which value is an estimate?	16. Which value is an approximation: 18.6 km or 19 km? Explain.
17. Option A costs \$12 for 3 items. Option B costs \$18.50 for 5 items. Which gives better value?	18. An excursion has 257 people. Each bus holds 45 people. Estimate how many buses are needed.
19. A calculator answer to 87.2×19.6 is 170.912. Is it reasonable? Explain briefly.	20. A square has area 264 cm^2 . Estimate its side length to 3 s.f.
21. A square has area 264 cm^2 . Estimate its perimeter to 3 s.f.	22. Classify the error: rounding an intermediate answer too early.
23. Give one reason why answers in real life are often estimated instead of exact.	24. Create one real-life situation where rounding to the nearest 10 is useful.

Challenge Yourself (8 questions)

1. Without a calculator, decide whether 987×123 is greater or less than 988×122 . Explain.	2. Estimate $\sqrt{50}$ to 3 significant figures using one iteration of averaging between 7 and $50/7$.
3. The number $21X09$ is 22 000 to 2 significant figures. If $21X09$ is a perfect square, find X.	4. A value rounded to 3 significant figures is 1.00. Give the possible range of actual values.
5. Explain why 999.9 to 3 significant figures is 1.00×10^3 , not simply 1000 if the degree of accuracy must be shown.	6. A rectangular field measures 27.04 m by 20.21 m. Estimate the area and state your rounding choices.
7. Store A: 500 GB drive for \$85.05 with 20% discount. Store B: same drive for \$78.05 with 10% discount. Estimate which is cheaper.	8. A container has 125 g of grains. Estimate how many grains fill the container if a 5 g sample has about 180 grains.

Teacher Notes and Lesson Use

Goal	Suggested use
For a 1.5-hour class	Assign Practice Sets A and B first. Then select Practice Set C or D depending on class needs. Use Review Exercise as timed practice.
For homework	Assign all practice sets except Challenge Yourself, then use the answers for self-marking.
For stronger students	Add Challenge Yourself. Ask students to explain estimation choices, not only compute answers.
For weaker students	Use the concept pages and worked examples first. Let students complete every alternate question before attempting all.

Common misconceptions to watch

- Dropping zeros that show significant figures, such as writing 27 instead of 27.0.
- Counting leading zeros in decimals as significant figures.
- Rounding intermediate working too early and causing follow-through errors.
- Confusing approximate values with estimated values.
- Using normal rounding when a real-life context requires rounding up, such as number of buses.

Answers: Practice Set A

1. 49 280	9. 78.5	17. \$11.00
2. 49 300	10. 78	18. 4.9 m
3. 49 000	11. 78.47	19. 10 cm
4. 50 000	12. 78.470	20. 6.5 kg
5. 698 350	13. 8	21. 6.49 kg
6. 698 400	14. 7.70	22. 0.05
7. 698 000	15. 7.7	23. 0.050
8. 700 000	16. 7.69715	24. 1000

Answers: Practice Set B

1. 5	9. 730	17. 22 000
2. 4	10. 504.0	18. 21 500
3. 8	11. 0.003020	19. 0.00099
4. 3	12. 6400	20. 0.0009877
5. 2	13. 6400	21. 6.00
6. 4	14. 10.0	22. 1 000 000
7. 4	15. 8.00	23. 999 900
8. 5	16. 0.0509	24. 0.0200

Answers: Practice Set C

1. 5 705 000	10. Examples: 50.35 s, 50.40 s, 50.44 s
2. 5 714 999	11. 3, 4, 5 or 6 depending on zeros indicated by accuracy
3. $7.35 \text{ cm} \leq x < 7.45 \text{ cm}$	12. Nearest 10 keeps tens place: $5192.3 \rightarrow 5190$, not 519.
4. $2.345 \text{ kg} \leq x < 2.355 \text{ kg}$	13. To 1 d.p., $26.97 \rightarrow 27.0$; the zero shows 1 decimal place.
5. 18.5 km	14. 0.0200 has 3 s.f.: 2, 0 and 0 after the non-zero digit.
6. 19.499... km / largest depends on unit precision	15. $\sqrt{105} \approx 10.247$; perimeter $\approx 41.0 \text{ cm}$
7. $224\,500 \text{ m}^2$	16. To reduce follow-through error.
8. $225\,499 \text{ m}^2$	17. 99.9%
9. Examples: 0.1196 cm, 0.1200 cm, 0.1204 cm	18. Rounding each percentage separately can make the total differ from 100%.

Answers: Practice Set D

1. About 40	10. About $40 \times \$5 = \$200$
2. About $80 \times 200 = 16\,000$	11. About $270 \div 20 \approx 13.5$ km/L
3. About $300 \div 30 = 10$	12. $32 \times 19 \approx 30 \times 20 = \$600$, nearest \$100 is \$600
4. About $5 \times 21 = 105$	13. $238 \div 4 \approx 60$ bottles
5. About S\$9	14. $65 \times 18 = 1170$ coins; value $\approx \$117$
6. About S\$2.50	15. About 37.5%
7. A: \$3 per 100 g; B $\approx$ \$2.98 per 100 g, so B slightly better	16. About 3 : 1
8. A: \$1.60 per 100 ml; B: \$1.56 per 100 ml, so B better	17. $\sqrt{50} \approx 7.1$
9. 220 people, 30 per bus $\rightarrow$ 8 buses	18. A rough estimate can show if the calculator result is far too large or small.

Answers: Review Exercise 3

1. 6 480 000	13. About $150 + 65 = 215$
2. 6 480 000	14. About $15 \times 65 = 975$
3. 96.5	15. About 10 km is the estimate.
4. 96	16. 19 km is an approximation if rounded from 18.6 km.
5. 0.0060	17. A: \$4 each; B: \$3.70 each, so B
6. 0.006020	18. $257 \div 45 \approx 6$ buses, but need 6 because 5 buses not enough? Actually $45 \times 5 = 225$, $45 \times 6 = 270$, so 6 buses.
7. 5	19. Not reasonable; estimate $87 \times 20 \approx 1740$, not 170.912.
8. 5	20. $\sqrt{264} \approx 16.2$ cm
9. 698	21. Perimeter ≈ 64.9 cm
10. 698.4	22. Follow-through error from early rounding
11. $48.55 \text{ kg} \leq x < 48.65 \text{ kg}$	23. Exact values may be unavailable, unnecessary or too slow.
12. 8500 to 9499	24. Example: estimating attendance, money, distance, or stock.

Answers: Challenge Yourself

1. $987 \times 123 = 987 \times (122 + 1)$; $988 \times 122 = (987 + 1) \times 122$. Difference = $987 - 122 = 865$, so 987×123 is greater.	5. 1000 alone may not show 3 significant figures; 1.00×10^3 shows 3 s.f.
2. $\sqrt{50} \approx 7.071$; to 3 s.f., 7.07.	6. $27.04 \times 20.21 \approx 27 \times 20 = 540 \text{ m}^2$
3. X = 9 because $21909 = 148^2$ and rounds to 22 000 to 2 s.f.	7. A $\approx 85 \times 0.8 = \$68$; B $\approx 78 \times 0.9 = \$70.20$, so A cheaper.
4. $0.995 \leq x < 1.005$	8. 180 grains per 5 g $\rightarrow$ 36 per g; 125 g $\rightarrow$ 4500 grains.