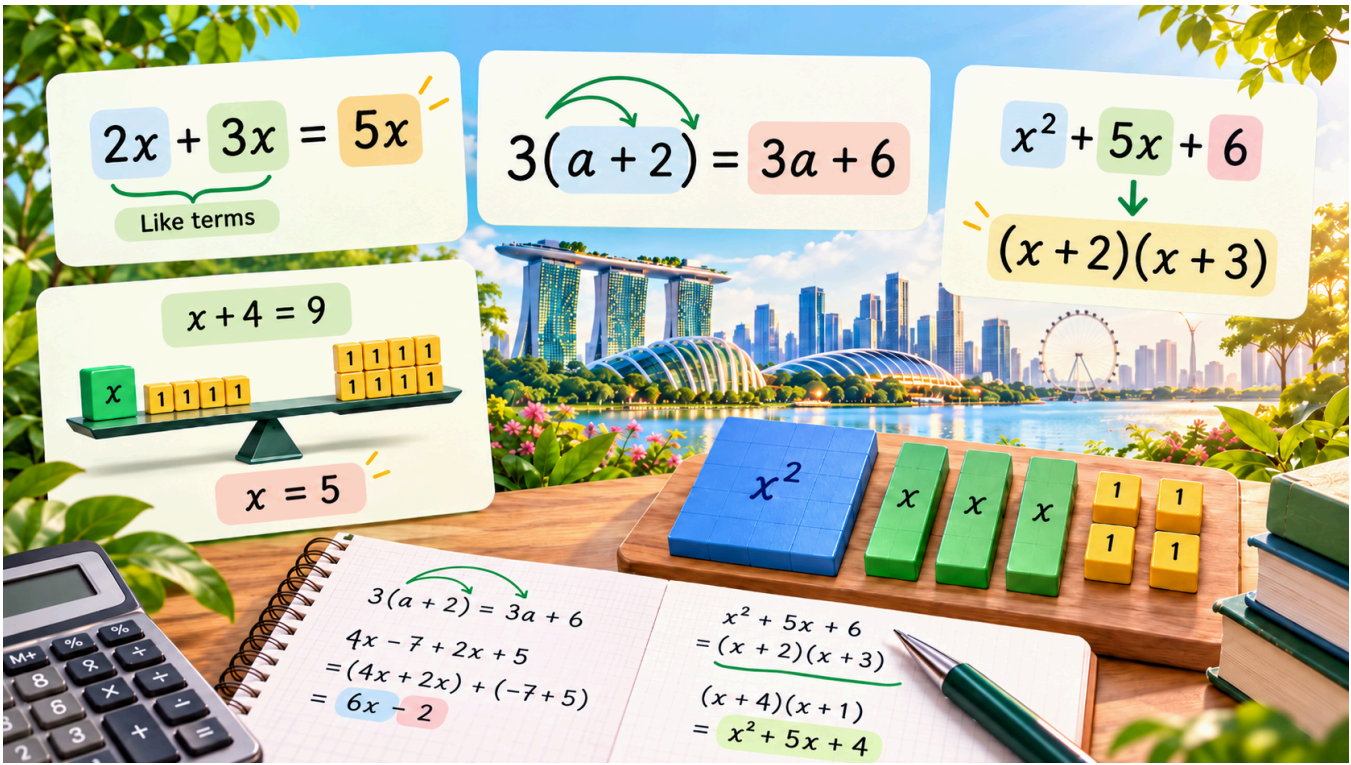


Basic Algebra and Algebraic Manipulation

Algebraic notation, variables, like terms, expansion, factorisation, fractional coefficients and mixed practice.



For students	For teachers	Practice load
Revise the full chapter with clear notes, worked examples, practice questions and answers.	Use as a full lesson, worksheet pack, projection reference or revision handout.	Expanded question bank designed for about 1.5 hours of student work, depending on class pace.

Built from the Chapter 4 topic flow: basic algebraic concepts and notation, algebraic expressions, evaluating expressions, like and unlike terms, addition and subtraction of linear terms, expansion, factorisation and linear expressions with fractional coefficients.

Chapter 4 Learning Map

Basic algebra is the bridge from arithmetic to general mathematical thinking. This chapter trains students to use letters meaningfully, simplify expressions correctly and manipulate expressions without changing their value.

Section	What students should master	Typical question types
4.1 Basic algebraic concepts and notation	Use letters for unknowns, generalised numbers and variables. Interpret notation such as $2n$, xy and x^2 .	Translate words to algebra; identify meaning of notation; evaluate simple expressions.
4.2 Addition and subtraction of linear terms	Recognise like and unlike terms and combine coefficients correctly, including negative coefficients.	Simplify $3x+5x$, $4a-7a$, $2x+3y-x$, and mixed expressions.
4.3 Expansion and factorisation	Use the distributive law to expand brackets and reverse the process to factorise expressions.	Expand $3(a+2)$; simplify after expansion; factorise $6x+12$ and $-4a+8$.
4.4 Fractional coefficients	Work with fractional coefficients and write algebraic expressions as single simplified fractions.	Simplify $\frac{1}{2}x + \frac{1}{3}x$; expand expressions with fractional multipliers.

90-minute practice plan

Time	Activity	Notes
10 min	Concept recap	Use pages 3-9 to revise notation, like terms, expansion and factorisation.
25 min	Practice Set A and B	Algebraic notation, evaluating expressions and simplifying like terms.
25 min	Practice Set C and D	Expansion, factorisation and fractional coefficients.
20 min	Review Exercise	Mixed questions under test-like conditions.
10 min	Check answers	Use answer pages to correct common mistakes.

1. Algebraic Letters and Notation

In algebra, letters can represent specific unknown numbers, generalised numbers or variables. Students must know what the letter represents before manipulating the expression.

Use of letter	Meaning	Example
Specific unknown	A letter stands for a number that is not yet known.	$x + 4 = 9$, so $x = 5$.
Generalised number	A letter represents any number in a pattern or rule.	$a + b = b + a$ for any numbers a and b .
Variable	A letter represents a quantity that can change.	$P = 4l$ for the perimeter of a square.

Common algebraic notation

Notation	Meaning	Possible mistake
$3x$	3 multiplied by x , or $x + x + x$.	Writing $3 + x$ instead of $3x$.
xy	x multiplied by y .	Thinking xy means a two-digit number.
x^2	x multiplied by x .	Thinking x^2 means $2x$.
$2(x + y)$	2 groups of $(x + y)$.	Multiplying only the first term.
$-x$	The negative of x .	Treating it as always a negative number, instead of the opposite of x .

2. Algebraic Expressions and Evaluation

An algebraic expression is made from numbers, letters and operation signs. To evaluate an expression, substitute the given values and calculate carefully.

Term	Coefficient	Variable part	Constant?
$2x$	2	x	No
$-3xy$	-3	xy	No
7	-	-	Yes
$\frac{1}{2}a$	$\frac{1}{2}$	a	No

Worked example 1: Evaluate expressions

Question	Working	Answer
Given $x = 5$ and $y = -3$, find $3x - 2y$.	$3(5) - 2(-3) = 15 + 6$	21
Given $a = 3$ and $b = -4$, find $ab - b$.	$3(-4) - (-4) = -12 + 4$	-8
Given $p = 2$ and $q = -1$, find $p^2 + 3q$.	$2^2 + 3(-1) = 4 - 3$	1

Trap to avoid

Mistake	Correction
Forgetting brackets with negative numbers: $3x - 2y = 15 - 6$ when $y = -3$.	Substitute as $3(5) - 2(-3)$. The second term becomes $+6$.

3. Like and Unlike Terms

Like terms contain the same variable part raised to the same powers. Only like terms can be added or subtracted directly.

Like terms	Why
$2x$ and $5x$	Both have the same variable part x .
$-3ab$ and $7ab$	Both have the same variable part ab .
4 and -9	Both are constants.
Unlike terms	Why
$2x$ and $2y$	Different variables.
x and x^2	Same letter but different powers.
$3ab$ and $3a$	Different variable parts.

Worked example 2: Combine like terms

Expression	Group like terms	Simplified answer
$3x + 5x$	$(3 + 5)x$	$8x$
$4a - 7a$	$(4 - 7)a$	$-3a$
$2x + 3y - x + 5y$	$(2x - x) + (3y + 5y)$	$x + 8y$
$6p - 4q - 2p + q$	$(6p - 2p) + (-4q + q)$	$4p - 3q$

4. Addition and Subtraction of Linear Terms

To add or subtract linear terms, add or subtract the coefficients and keep the variable part unchanged. The same rule applies when coefficients are negative.

Rule	Example
Add coefficients of like terms.	$5x + 2x = 7x$
Subtract coefficients of like terms.	$8a - 3a = 5a$
Keep unlike terms separate.	$2x + 3y$ cannot be simplified further.
Be careful with negative signs.	$-2x + (-3x) = -5x$; $5x - (-2x) = 7x$

Worked example 3: Simplify expressions with two variables

Question	Working	Answer
$5x + 2y - 3x + 4y$	$(5x - 3x) + (2y + 4y)$	$2x + 6y$
$-4a + 7b + 9a - 2b$	$(-4a + 9a) + (7b - 2b)$	$5a + 5b$
$3m - 2n - 5m + n$	$(3m - 5m) + (-2n + n)$	$-2m - n$

5. Expansion Using the Distributive Law

Expansion changes a product into a sum or difference. The factor outside the bracket must be multiplied by every term inside the bracket.

Distributive law
$a(b + c) = ab + ac$
$a(b - c) = ab - ac$

Question	Working	Answer
$3(a + 2)$	$3 \times a + 3 \times 2$	$3a + 6$
$-2(x + y)$	$(-2)x + (-2)y$	$-2x - 2y$
$4(2p - 3)$	$4 \times 2p - 4 \times 3$	$8p - 12$
$-5(3x - 2)$	$(-5)(3x) + (-5)(-2)$	$-15x + 10$

Common mistake

Incorrect	Correct
$3(a + 2) = 3a + 2$	$3(a + 2) = 3a + 6$. Multiply both terms by 3.

6. Factorisation of Linear Expressions

Factorisation is the reverse of expansion. It expresses an algebraic expression as a product of factors.

Expanded expression	Common factor	Factorised form
$6x + 12$	6	$6(x + 2)$
$15a - 20$	5	$5(3a - 4)$
$8xy + 12x$	$4x$	$4x(2y + 3)$
$-6x - 15$	-3	$-3(2x + 5)$

Extracting a negative common factor

Expression	Factorised form	Check by expansion
$-x - y$	$-(x + y)$	$-x - y$
$-6a + 9$	$-3(2a - 3)$	$-6a + 9$
$-4p - 12q$	$-4(p + 3q)$	$-4p - 12q$

7. Fractional Coefficients

A linear expression may have fractional coefficients. To simplify, group like terms and use equivalent fractions when necessary.

Question	Working	Answer
$\frac{1}{2}x + \frac{1}{3}x$	$(\frac{3}{6} + \frac{2}{6})x$	$\frac{5}{6}x$
$\frac{3}{4}a - \frac{1}{2}a$	$(\frac{3}{4} - \frac{2}{4})a$	$\frac{1}{4}a$
$\frac{2}{3}p + \frac{1}{6}p - p$	$(\frac{4}{6} + \frac{1}{6} - \frac{6}{6})p$	$-\frac{1}{6}p$
$\frac{1}{2}(4x - 6)$	$2x - 3$	$2x - 3$

Worked example 4: Expand and simplify

Question	Working	Answer
$\frac{1}{3}(6x - 9) + 2x$	$2x - 3 + 2x$	$4x - 3$
$\frac{1}{2}(8a + 4) - 3a$	$4a + 2 - 3a$	$a + 2$
$\frac{3}{4}(8p - 12) + p$	$6p - 9 + p$	$7p - 9$

8. Common Algebra Traps

Trap	Why it matters	Safer habit
Treating unlike terms as like terms	$2x + 3y$ is not $5xy$ or $5x$.	Check the variable part before combining.
Forgetting brackets with negatives	Substitution errors happen when $x = -3$ or $y = -2$.	Write brackets before calculating.
Multiplying only one term in a bracket	$3(a + 2)$ is not $3a + 2$.	Distribute to every term.
Extracting the wrong common factor	A factorised answer may not expand back correctly.	Expand your answer to check.
Losing signs when factorising negatives	$-x - y$ is $-(x + y)$, not $-(x - y)$.	Check by expanding the factorised form.
Ignoring fractional coefficients	$1/2x + 1/3x$ is not $2/5x$.	Use common denominators.

Mini-check before practice

Question	Answer
Can $4x + 3y$ be simplified further?	No, x and y are unlike terms.
What is $2(a + 5)$?	$2a + 10$.
What is the common factor of $8x + 12$?	4.
What is $1/2 x + 1/4 x$?	$3/4 x$.

Practice Set A: Algebraic notation and evaluation (24 questions)

1. Write an algebraic expression for the sum of x and 7.	13. Given $x = 5$, evaluate $x + 4$.
2. Write an algebraic expression for 5 less than y .	14. Given $y = -2$, evaluate $3y$.
3. Write an algebraic expression for 3 times a number n .	15. Given $a = 3$ and $b = 4$, evaluate $ab + 2$.
4. Write an algebraic expression for half of p .	16. Given $p = -1$, evaluate $p^2 + 5p$.
5. Write an expression for the product of a and b .	17. Given $x = 2$ and $y = -3$, evaluate $3x - 2y$.
6. Write an expression for twice the sum of x and 4.	18. Given $a = -4$, evaluate $2(a + 7)$.
7. Write an expression for 6 more than $3k$.	19. Given $n = 6$, evaluate $n/2 + 5$.
8. Write an expression for the difference between m and 9.	20. Given $q = -5$, evaluate $q^2 - q$.
9. State the meaning of $4x$.	21. Use letters to represent: the cost of 8 pens at $\$p$ each.
10. State the meaning of ab .	22. Use letters to represent: the perimeter of a square of side s .
11. State the meaning of x^2 .	23. Use letters to represent: the total age of Ali, age a , and Ben, age b .
12. State the meaning of $3(x + 2)$.	24. A number is multiplied by 3 and then 7 is added. Write an expression.

Practice Set B: Like terms and simplification (24 questions)

1. Simplify $3x + 5x$.	13. Simplify $3a + 4 - a + 7$.
2. Simplify $9a - 4a$.	14. Simplify $10p - 8 + 2p - 5$.
3. Simplify $7p + p$.	15. Simplify $-2x + 9 - 5x + 1$.
4. Simplify $12m - 15m$.	16. Simplify $4a - 3b - 7a + 8b$.
5. Simplify $-3x + 8x$.	17. Simplify $x + x + x + 5$.
6. Simplify $-4a - 6a$.	18. Simplify $2r - r + 6r$.
7. Simplify $5y - (-2y)$.	19. Simplify $7 - 3q + 9q - 2$.
8. Simplify $-7p + 3p$.	20. Simplify $-5m + 2n + 8m - 6n$.
9. Simplify $2x + 3y - x$.	21. Are $2xy$ and $5xy$ like terms?
10. Simplify $5a + 2b - 3a + b$.	22. Are $3x$ and $3x^2$ like terms?
11. Simplify $8m - 2n - 3m + 5n$.	23. Are $-4ab$ and $9ba$ like terms?
12. Simplify $-4x + 6y + 9x - y$.	24. Explain why $2x + 3y$ cannot be simplified further.

Practice Set C: Expansion and factorisation (24 questions)

1. Expand $2(x + 5)$.	13. Factorise $6x + 12$.
2. Expand $3(a + 4)$.	14. Factorise $10a + 15$.
3. Expand $5(p - 2)$.	15. Factorise $8p - 20$.
4. Expand $-2(x + 3)$.	16. Factorise $12m + 18n$.
5. Expand $-4(y - 5)$.	17. Factorise $15xy - 10x$.
6. Expand $6(2m + 1)$.	18. Factorise $-x - 4$.
7. Expand $3(2a - 7)$.	19. Factorise $-6a - 12$.
8. Expand $-5(3x - 2)$.	20. Factorise $-9p + 3$.
9. Expand and simplify $2(x + 3) + 4x$.	21. Factorise $14x + 21$.
10. Expand and simplify $3(a - 2) + 5a$.	22. Factorise $20ab - 25a$.
11. Expand and simplify $-2(p + 4) + 7p$.	23. Expand then simplify $2(3x + 4) - x$.
12. Expand and simplify $4(2x - 1) - 3x$.	24. Factorise completely $18x - 24$.

Practice Set D: Fractional coefficients (18 questions)

1. Simplify $\frac{1}{2}x + \frac{1}{2}x$.	10. Expand $\frac{1}{3}(6a - 9)$.
2. Simplify $\frac{1}{2}x + \frac{1}{3}x$.	11. Expand $\frac{1}{4}(8p + 12)$.
3. Simplify $\frac{3}{4}a - \frac{1}{2}a$.	12. Expand $-\frac{1}{2}(6y - 10)$.
4. Simplify $\frac{2}{3}p + \frac{1}{6}p$.	13. Expand and simplify $\frac{1}{2}(4x - 6) + x$.
5. Simplify $\frac{5}{6}y - \frac{1}{3}y$.	14. Expand and simplify $\frac{1}{3}(9a + 6) - 2a$.
6. Simplify $\frac{1}{4}m + \frac{3}{8}m$.	15. Expand and simplify $\frac{1}{4}(8p - 12) + 3p$.
7. Simplify $\frac{2}{5}q - \frac{1}{10}q$.	16. Write $\frac{1}{2}x + \frac{1}{3}$ as a single fraction.
8. Simplify $\frac{3}{2}x - \frac{1}{2}x$.	17. Write $(x + 2)/3 + x/6$ as a single fraction.
9. Expand $\frac{1}{2}(4x + 6)$.	18. Write $(2a - 5)/4 - a/2$ as a single fraction.

Review Exercise 4: Mixed practice (24 questions)

1. Write an expression for 4 more than twice x .	13. Factorise $9x + 18$.
2. Write an expression for the total cost of 7 books at \$ b each.	14. Factorise $16a - 24$.
3. Given $x = -3$, evaluate $2x + 5$.	15. Factorise $-5p - 10$.
4. Given $a = 4$ and $b = -2$, evaluate $ab - 3b$.	16. Factorise $12xy + 18x$.
5. State whether $5x$ and $-2x$ are like terms.	17. Simplify $\frac{1}{2}x + \frac{3}{4}x$.
6. State whether $3a$ and $3b$ are like terms.	18. Simplify $\frac{5}{6}a - \frac{1}{3}a$.
7. Simplify $7x + 4x - 2x$.	19. Expand $\frac{1}{2}(8m - 4)$.
8. Simplify $3a - 5b + 4a + 2b$.	20. Expand and simplify $\frac{1}{3}(6p + 9) - p$.
9. Simplify $-6p + 9 - 4p - 2$.	21. A number is x . Write an expression for the next odd number after x if x is odd.
10. Expand $5(x + 3)$.	22. A rectangle has length l and breadth b . Write its perimeter.
11. Expand $-3(a - 4)$.	23. Explain why $2(x + 3) = 2x + 6$.
12. Expand $2(3p - 5) + p$.	24. Explain how to check whether factorisation is correct.

Challenge Yourself (8 questions)

1. Find the product of $(a-b)$, $(b-c)$, $(c-d)$, ..., $(y-z)$, $(z-a)$ without expanding every factor.	5. Without expanding fully, decide whether $3(x+4) + 2(x+4)$ equals $5(x+4)$. Explain.
2. Cheryl's trick: choose a two-digit number greater than 20, add the digits, subtract the sum from the original number. Explain why the final answer is always a multiple of 9.	6. Factorise $12ab - 18a + 6ac$ completely.
3. Show that a two-digit number with tens digit x and ones digit y can be written as $10x + y$.	7. Simplify $\frac{1}{2}(6x - 4) - \frac{1}{3}(9x + 6)$.
4. The digits of a two-digit number are swapped. Express the difference between the original number and the swapped number.	8. Create your own expression that expands to $8x - 12$ and has a negative factor outside the bracket.

Teacher Notes and Lesson Use

Goal	Suggested use
For a 1.5-hour class	Assign Practice Set A and B first. Then select Practice Set C or D depending on class needs. Use Review Exercise as timed practice.
For homework	Assign the full practice sets except Challenge Yourself, then use the answers for self-marking.
For stronger students	Add Challenge Yourself. Ask students to justify algebraic equivalence, not only compute answers.
For weaker students	Use the concept pages first. Let students complete every alternate question before attempting all.

Common misconceptions to watch

Misconception
Combining unlike terms, such as writing $2x + 3y = 5xy$.
Forgetting to multiply every term when expanding brackets.
Dropping negative signs during subtraction or factorisation.
Thinking x^2 means $2x$.
Adding fractions without common denominators.

Answers: Practice Set A

1. $x + 7$	9. 4 times x	17. 12
2. $y - 5$	10. a times b	18. 6
3. $3n$	11. x times x	19. 8
4. $p/2$	12. 3 times $(x + 2)$	20. 30
5. ab	13. 9	21. $8p$
6. $2(x + 4)$	14. -6	22. $4s$
7. $3k + 6$	15. 14	23. $a + b$
8. $m - 9$	16. 6	24. $3x + 7$

Answers: Practice Set B

1. $8x$	9. $x + 3y$	17. $3x + 5$
2. $5a$	10. $2a + 3b$	18. $7r$
3. $8p$	11. $5m + 3n$	19. $4 + 6q$
4. $-3m$	12. $5x + 5y$	20. $3m - 4n$
5. $5x$	13. $2a + 11$	21. Yes
6. $-10a$	14. $12p - 13$	22. No
7. $7y$	15. $-7x + 10$	23. Yes
8. $-4p$	16. $-3a + 5b$	24. Different variables, so they are unlike terms.

Answers: Practice Set C

1. $2x + 10$	9. $6x + 6$	17. $5x(3y - 2)$
2. $3a + 12$	10. $8a - 6$	18. $-(x + 4)$
3. $5p - 10$	11. $5p - 8$	19. $-6(a + 2)$
4. $-2x - 6$	12. $5x - 4$	20. $-3(3p - 1)$
5. $-4y + 20$	13. $6(x + 2)$	21. $7(2x + 3)$
6. $12m + 6$	14. $5(2a + 3)$	22. $5a(4b - 5)$
7. $6a - 21$	15. $4(2p - 5)$	23. $5x + 8$
8. $-15x + 10$	16. $6(2m + 3n)$	24. $6(3x - 4)$

Answers: Practice Set D

1. x	7. $\frac{3}{10}q$	13. $3x - 3$
2. $\frac{5}{6}x$	8. x	14. $a + 2$
3. $\frac{1}{4}a$	9. $2x + 3$	15. $5p - 3$
4. $\frac{5}{6}p$	10. $2a - 3$	16. $(3x + 2)/6$
5. $\frac{1}{2}y$	11. $2p + 3$	17. $(x + 1)/2$
6. $\frac{5}{8}m$	12. $-3y + 5$	18. $-5/4$

Answers: Review Exercise 4

1. $2x + 4$	9. $-10p + 7$	17. $\frac{5}{4}x$
2. $7b$	10. $5x + 15$	18. $\frac{1}{2}a$
3. -1	11. $-3a + 12$	19. $4m - 2$
4. -2	12. $7p - 10$	20. $p + 3$
5. Yes	13. $9(x + 2)$	21. $x + 2$
6. No	14. $8(2a - 3)$	22. $2l + 2b$
7. $9x$	15. $-5(p + 2)$	23. Multiply both x and 3 by 2 .
8. $7a - 3b$	16. $6x(2y + 3)$	24. Expand the factorised answer and compare.

Answers: Challenge Yourself

1.	The product includes both (a-a) somewhere in the cycle after multiplying all consecutive differences? It contains a zero factor if any repeated factor is present; otherwise leave as product form. Discuss carefully.
2.	Let the number be $10x + y$. Sum of digits = $x + y$. Difference = $10x + y - (x + y) = 9x$, a multiple of 9 .
3.	$10x + y$.
4.	Original - swapped = $(10x + y) - (10y + x) = 9x - 9y = 9(x - y)$.
5.	Yes. $3(x+4) + 2(x+4) = (3+2)(x+4) = 5(x+4)$.
6.	$6a(2b - 3 + c)$.
7.	$3x - 2 - 3x - 2 = -4$.
8.	Example: $-4(-2x + 3) = 8x - 12$.