

Linear Equations

Linear equations, fractional equations, word problems and mathematical formulae.

Balance

Keep both sides balanced when we do the same operation.

$x + 5 = 12$

Inverse operations

$x + 5 = 12$
 $-5 \quad -5$
 $x = 7$

Inverse operations

$3x - 4 = 11$
 $+4 \quad +4$
 $3x = 15$
 $\div 3 \quad \div 3$
 $x = 5$

Fractional equation

$\frac{x}{2} + 3 = 9$
 $-3 \quad -3$
 $\frac{x}{2} = 6$
 $\times 2 \quad \times 2$
 $x = 12$

Word problem

3 identical pens and \$2 cost \$11.

Let p be the cost of one pen.

$3p + 2 = 11$

Formula

$A = l \times b$

$l = 7$
 $b = 4$
 $\downarrow$
 $A = 7 \times 4$
 $A = 28$

$x + 5 = 12$
 $x = 12 - 5$
 $x = 7$

For students	For teachers	Practice load
Revise the full chapter with notes, worked examples, practice questions and answers.	Use as a full lesson, worksheet pack, projection reference or revision handout.	Expanded practice designed for about 1.5 hours of student work, depending on class pace.

Built from the Chapter 5 topic flow: linear equations, equivalent equations, balancing method, fractional coefficients, fractional equations, applications in real-world contexts, evaluating formulae and constructing formulae.

Chapter 5 Learning Map

Linear equations help us find unknown values when two expressions are equal. This chapter also connects algebra to word problems and mathematical formulae.

Section	What students should master	Typical question types
5.1 Linear equations	Understand equations, equivalent equations and balancing operations.	Solve equations with brackets, variables on both sides and integer coefficients.
5.2 Fractional equations	Solve equations with fractional coefficients and equations with variable denominators.	Clear denominators, state restrictions and check solutions.
5.3 Applications	Translate real-world contexts into equations.	Age, number, cost, distance, speed and model-to-algebra questions.
5.4 Formulae	Substitute into formulae and construct formulae from descriptions.	Evaluate $V = lbh$, $A = \frac{1}{2}bh$, $F = ma$, $s = ut + \frac{1}{2}at^2$, and build formulae.

90-minute practice plan

Time	Activity	Notes
10 min	Concept recap	Use pages 3–7 to revise key ideas.
25 min	Practice Set A and B	Linear equations and fractional equations.
25 min	Practice Set C and D	Applications and formulae.
20 min	Review Exercise	Mixed questions under test-like conditions.
10 min	Check answers	Use selected explanations for common mistakes.

1. Concept of Equation

An equation is a mathematical statement that two expressions have equal value. The equal sign means both sides must have the same value.

Expression: $x + 2$ is an expression. It does not have an equal sign.

Equation: $x + 2 = 5$ is an equation. It can be true or false depending on the value of x .

Solution: $x = 3$ is the solution of $x + 2 = 5$ because $3 + 2 = 5$.

Key distinction

Term	Meaning	Example
Expression	A combination of numbers, variables and operations.	$2x + 3$
Equation	Two expressions connected by an equal sign.	$2x + 3 = 9$
Linear equation	An equation where the highest power of the variable is 1.	$3x - 4 = 11$
Equivalent equations	Equations with the same solution.	$x + 5 = 12$ and $x = 7$

2. Solving Linear Equations

To solve an equation, we isolate the unknown while keeping both sides balanced. Whatever operation is done to one side must be done to the other side.

Worked example 1:

$$\begin{aligned}x + 5 &= 12 \\x + 5 - 5 &= 12 - 5 \\x &= 7\end{aligned}$$

Check: $7 + 5 = 12$.

Worked example 2:

$$\begin{aligned}3x - 4 &= 11 \\3x - 4 + 4 &= 11 + 4 \\3x &= 15 \\x &= 5\end{aligned}$$

Check: $3(5) - 4 = 11$.

Common mistake

Students sometimes move terms across the equal sign without understanding the balancing operation. It is safer to think: do the same operation to both sides.

3. Equations with Variables on Both Sides

When a variable appears on both sides, collect the variable terms on one side and constants on the other side.

Worked example 3:

$$\begin{aligned}5x + 4 &= 2x - 8 \\5x - 2x + 4 &= 2x - 2x - 8 \\3x + 4 &= -8 \\3x &= -12 \\x &= -4\end{aligned}$$

Worked example 4:

$$\begin{aligned}2(y - 4) &= 5y - 3 \\2y - 8 &= 5y - 3 \\-8 + 3 &= 5y - 2y \\-5 &= 3y \\y &= -\frac{5}{3}\end{aligned}$$

4. Fractional Coefficients and Fractional Equations

Fractions must be written clearly. We clear denominators by multiplying both sides by the lowest common multiple of the denominators.

Worked example 5: Fractional coefficient

$$\begin{aligned}\frac{x}{2} + 3 &= 9 \\ \frac{x}{2} &= 6 \\ x &= 12\end{aligned}$$

Worked example 6: Clear denominators

$$\begin{aligned}\frac{x+1}{2} + \frac{x-1}{3} &= 5 \\ 6\left(\frac{x+1}{2}\right) + 6\left(\frac{x-1}{3}\right) &= 30 \\ 3(x+1) + 2(x-1) &= 30 \\ 3x + 3 + 2x - 2 &= 30 \\ 5x + 1 &= 30 \\ x &= \frac{29}{5}\end{aligned}$$

Restriction

For equations with variable denominators, the denominator cannot be zero. For example, in $\frac{1}{x-3}$, $x \neq 3$.

5. Applications of Linear Equations

Many real-world problems can be solved by defining a variable and forming an equation.

Worked example 7: Consecutive even numbers

The sum of three consecutive even numbers is 60. Let the smallest even number be n . The numbers are n , $n + 2$ and $n + 4$.

$$n + (n + 2) + (n + 4) = 60$$

$$3n + 6 = 60$$

$$3n = 54$$

$$n = 18$$

The numbers are 18, 20, 22.

Worked example 8: Word problem to equation

Three identical pens and a \$2 delivery charge cost \$11. Let p be the cost of one pen.

$$3p + 2 = 11, \quad 3p = 9, \quad p = 3.$$

Each pen costs \$3.

6. Mathematical Formulae

A formula shows the relationship between variables. We can substitute values to evaluate a formula or rearrange a formula to find an unknown.

Worked example 9: Evaluating formulae

$$V = lbh$$

If $l = 7$, $b = 4$, $h = 3$,

$$V = 7 \times 4 \times 3 = 84.$$

Worked example 10: Formula with a fraction

$$A = \frac{1}{2}bh$$

If $A = 84$ and $b = 12$,

$$84 = \frac{1}{2}(12)h$$

$$84 = 6h$$

$$h = 14$$

Constructing a formula

If n pens cost $\$p$ each and delivery costs $\$2$, then the total cost is

$$C = np + 2.$$

Practice Set A: Linear equations (24 questions)

1. Solve $x + 8 = 15$.

2. Solve $x + 9 = -5$.

3. Solve $x - 5 = 17$.

4. Solve $y - 7 = -3$.

5. Solve $4x = 28$.

6. Solve $-2k = -14$.

7. Solve $\frac{x}{3} = 11$.

8. Solve $\frac{p}{-7} = -12$.

9. Solve $3x + 4 = 19$.

10. Solve $2y - 5 = 13$.

11. Solve $5a + 7 = 42$.

12. Solve $6m - 11 = 25$.

13. Solve $4x + 3 = 2x + 15$.

14. Solve $7y - 5 = 3y + 11$.

15. Solve $9p + 4 = 5p - 16$.

16. Solve $8 - 3t = 2t + 18$.

17. Solve $3(x + 2) = 21$.

18. Solve $4(y - 5) = 12$.

19. Solve $2(3x - 1) = 22$.

20. Solve $5(2a + 1) = 35$.

21. Solve $6 - 2(x + 3) = 12$.

22. Solve $3(2p - 5) = p + 10$.

23. Solve $4(x - 2) + 3 = 2x + 7$.

24. Solve $5 - 2(3y + 1) = 15$.

Practice Set B: Fractional coefficients and fractional equations (24 questions)

The questions in this section are spaced more widely so that the fractions remain readable.

1. Solve $\frac{x}{2} + 3 = 9$.

2. Solve $\frac{x}{3} - 2 = 4$.

3. Solve $\frac{2x}{5} + 1 = 7$.

4. Solve $\frac{3x - 1}{4} = 5$.

5. Solve $\frac{x}{2} + x = 10$.

6. Solve $\frac{x}{4} + 3 = \frac{x}{2} + 5$.

7. Solve $\frac{2x + 1}{3} = x + 5$.

8. Solve $\frac{x - 1}{5} + x + 3 = 4$.

9. Solve $\frac{3x - 1}{2} = x - 4$.

10. Solve $\frac{x + 2}{3} = 3x - 2$.

11. Solve $\frac{9}{2x - 5} = 3$.

12. Solve $\frac{y + 4}{2y - 3} = 2$.

Practice Set B continued

13. State the restriction for $\frac{1}{x-3}$.

14. State the restriction for $\frac{5}{2x+1}$.

15. Solve $\frac{6}{x} = 3$.

16. Solve $2 + \frac{4}{x-1} = 6$.

17. Solve $\frac{5}{x+2} = 1$.

18. Solve $\frac{3}{2x-1} = 1$.

19. Explain why $x = 2$ is not allowed in $\frac{1}{x-2}$.

20. Explain why fractional equations must be checked after solving.

21. Solve $\frac{x+1}{2} + \frac{x-1}{3} = 5$.

22. Solve $\frac{2x-3}{4} - \frac{x+1}{6} = 1$.

23. Solve $\frac{x}{5} + x = 10$.

24. Solve $\frac{3x}{4} - x = 5$.

Practice Set C: Applications of linear equations (18 questions)

1. The sum of three consecutive even numbers is 60. Find the numbers.
2. The sum of two numbers, one of which is 5 times as large as the other, is 24. Find the two numbers.
3. A score is 15 marks more than another score. If twice the smaller score is 4 less than the larger score, find the smaller score.
4. A man is 5 times as old as his son. In 10 years' time, the sum of their ages will be 76. Find their present ages.
5. A delivery has a fixed cost of \$2 plus p dollars per pen. Three pens cost \$11. Form an equation and find p .
6. A taxi fare is \$3.50 plus \$0.80 per km. If the fare is \$18.70, find the distance travelled.
7. A rectangle has length $x + 5$ cm and breadth x cm. Its perimeter is 50 cm. Find x .
8. A number is tripled and then increased by 8. The result is 41. Find the number.
9. Ali walked for 45 minutes and ran for 30 minutes. His running speed is twice his walking speed. Total distance is 9 km. Find his walking speed.
10. A school has 12 more boys than girls. If there are 236 students, find the number of boys.
11. The numerator of a fraction is 5 less than its denominator. If 1 is added to both, the new fraction is $\frac{2}{3}$. Find the fraction.
12. A two-digit number has ones digit 2.5 times its tens digit. If the digits are reversed, the new number is 27 more. Find the number.
13. A bag of coins has twice as many 20-cent coins as 50-cent coins. The total value is \$5.40. Find the number of each coin.

14. A train travels from Town A to Town B at 40 km/h and returns at 60 km/h. The total time is 5 hours. Find the one-way distance.

15. Three consecutive odd numbers have a sum of 93. Find the numbers.

16. A shop sells notebooks at \$ n each. Four notebooks and a \$3 delivery charge cost \$19. Find n .

17. A triangle has angles x° , $2x^\circ$, and $(x + 20)^\circ$. Find x .

18. A father is 4 times as old as his child. In 6 years, he will be 3 times as old. Find their present ages.

Practice Set D: Mathematical formulae (18 questions)

Formula questions are kept in spacious one-column rows because many contain fractions, variables and conditions.

1. Use $V = lbh$ to find V when $l = 7$, $b = 4$, $h = 3$.

2. Use $A = \frac{1}{2}bh$ to find A when $b = 12$, $h = 9$.

3. Use $F = ma$ to find F when $m = 6$, $a = 5$.

4. Use $s = ut + \frac{1}{2}at^2$ to find s when $u = 2$, $a = 4$, $t = 3$.

5. If $V = lbh$, find h when $V = 240$, $l = 6$, $b = 5$.

6. If $A = \frac{1}{2}bh$, find h when $A = 84$, $b = 12$.

7. If $F = ma$, find a when $F = 72$, $m = 9$.

8. If $y = mx + c$, find x when $y = 17$, $m = 3$, $c = 2$.

9. Find a formula for the sum S of three consecutive even numbers.

Practice Set D continued

10. Find a formula for the sum S of four consecutive odd numbers.

11. Find S if $S = n + (n + 2) + (n + 4)$ and the greatest number is 101.

12. Find S for four consecutive odd numbers if the greatest odd number is 57.

13. Convert $C = \frac{5}{9}(F - 32)$ to find C when $F = 68$.

14. Use $F = \frac{9}{5}C + 32$ to find F when $C = -5$.

15. An object falls with $s = \frac{1}{2}gt^2$. Find s when $g = 9.8$, $t = 4$.

16. If $p = \frac{a + b}{2}$, find b when $p = 9$, $a = 7$.

17. Construct a formula for the cost C of n pens at \$ p each plus a \$2 delivery fee.

18. Construct a formula for the perimeter P of a rectangle with length l and breadth b .

Review Exercise 5: Mixed practice (24 questions)

1. Solve $x - 1 = \frac{1}{2}x$.

2. Solve $2(x - 1) + 3(x + 1) = 4(x + 4)$.

3. Solve $2y - 7 = 5y - 41$.

4. Solve $3x - 5 = 0.5x$.

5. Solve $\frac{2y + 7}{4} = 12$.

6. Solve $\frac{4y - 1}{5} = 5$.

7. Solve $\frac{a + 1}{4} + \frac{a - 1}{2} = 4$.

8. Solve $\frac{b - 4}{3} + \frac{2b + 1}{6} = \frac{5b - 1}{2}$.

9. Solve $\frac{2c - 5}{9} = \frac{c - 1}{6} - \frac{c + 3}{2}$.

10. Solve $\frac{x + 3}{2x - 1} = 4$.

11. If $3(x - 1) - 5(x - 4) = 8$, find x .

12. If $2y + 5 = 17$, find the value of $4y - 3$.

Review Exercise 5 continued

13. If $V = \frac{1}{3}\pi r^2 h$, find V when $r = 7$, $h = 9$. Use $\pi = \frac{22}{7}$.

14. If $A = \frac{1}{2}bh$, find h when $A = 84$ and $b = 12$.

15. Find two consecutive odd numbers with sum 64.

16. A man travels from Town A to Town B at 40 km/h and returns at 60 km/h. Total time is 5 hours. Find the one-way distance.

17. The numerator of a fraction is 2 less than its denominator. If 3 is subtracted from numerator and denominator, the new fraction is $\frac{3}{5}$. Find the fraction.

18. The sum of the ages of Iman and his cousin is 38. Seven years ago, Iman was thrice as old as his cousin. Find Iman's present age.

19. Object A is 5 kg heavier than Object B. Object C is twice Object A. Total mass is 255 kg. Find Object C.

20. Nadia is twice as old as Raju and half as old as Bernard. In 22 years' time, Bernard will be twice as old as Raju. Find Nadia's present age.

21. Construct a formula for the sum of five consecutive integers.

22. Use $U = \frac{1}{2}(r + h)$ to find h when $U = 16\frac{1}{2}$ and $r = 2$.

23. If $\frac{m+n}{n} = 5$, find m when $n = 6$.

24. Solve $\frac{4}{x-2} + \frac{1}{x} = 3$.

Challenge Yourself

1. Find a formula, in terms of n , for the sum of three consecutive odd numbers. Hence find the sum if the greatest odd number is a multiple of 3.

2. If x, y, z are positive integers such that $x < y < z$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find possible values.

3. There is an interesting anomaly involving $\frac{16}{64} = \frac{1}{4}$. Explain why normal cancellation is not allowed, then find another two-digit fraction with the same property.

4. A formula is $S = 5n + 10$. Find n when $S = 105$, and explain what the solution means in context.

5. A number is increased by 20%, then decreased by 20%. Form an equation to compare the final value with the original.

6. A rectangle has perimeter P and length l . Construct a formula for its breadth b , then find b when $P = 50, l = 14$.

Teacher Notes and Lesson Use

Goal	Suggested use
For a 1.5-hour class	Assign Practice Set A and selected questions from Practice Set B. Use Practice Set C or D depending on the class focus.
For homework	Assign selected questions from each practice set and use the answers for self-marking.
For stronger students	Add Challenge Yourself and require written explanations.
For weaker students	Use worked examples first. Then let students attempt alternate questions before attempting a full set.

Common misconceptions to watch

- Treating $x + 5 = 12$ as if the $+5$ simply jumps across the equal sign.
- Forgetting to perform the same operation on both sides.
- Losing negative signs when expanding brackets.
- Clearing denominators incorrectly in fractional equations.
- Forgetting denominator restrictions in fractional equations.
- Writing ambiguous fractions such as $5/6x$ instead of proper mathematical notation.

Answers: Practice Set A

- | | | |
|--------|--------|--------|
| 1. 7 | 9. 5 | 17. 5 |
| 2. -14 | 10. 9 | 18. 8 |
| 3. 22 | 11. 7 | 19. 4 |
| 4. 4 | 12. 6 | 20. 3 |
| 5. 7 | 13. 6 | 21. -6 |
| 6. 7 | 14. 4 | 22. 5 |
| 7. 33 | 15. -5 | 23. 6 |
| 8. 84 | 16. -2 | 24. -6 |

Answers: Practice Set B

- | | |
|-------------------|---------------------------|
| 1. 12 | 14. $x \neq -\frac{1}{2}$ |
| 2. 18 | 15. 2 |
| 3. 15 | 16. 2 |
| 4. $\frac{7}{20}$ | 17. 3 |
| 5. $\frac{3}{8}$ | 18. 2 |
| 6. 8 | 19. denominator becomes 0 |
| 7. -14 | 20. check restrictions |
| 8. $\frac{8}{3}$ | 21. $\frac{31}{5}$ |
| 9. $-\frac{7}{8}$ | 22. $\frac{19}{8}$ |
| 10. $\frac{8}{7}$ | 23. $\frac{25}{3}$ |
| 11. 4 | 24. -20 |
| 12. 10 | |
| 13. $x \neq 3$ | |

Answers: Practice Set D

- | | |
|-----------------|-------------------|
| 1. 84 | 10. $S = 4n + 16$ |
| 2. 54 | 11. 297 |
| 3. 30 | 12. 216 |
| 4. 24 | 13. 20 |
| 5. 8 | 14. 23 |
| 6. 14 | 15. 78.4 |
| 7. 8 | 16. 11 |
| 8. 5 | 17. $C = np + 2$ |
| 9. $S = 3n + 6$ | 18. $P = 2l + 2b$ |

Selected Answers and Hints

Practice Set C

1. 18, 20, 22.
2. 4 and 20.
5. $p = 3$.
7. $x = 10$.
9. walking speed = 6 km/h.
11. $\frac{10}{15} = \frac{2}{3}$.
13. 6 fifty-cent coins and 12 twenty-cent coins.
14. 120 km.
15. 29, 31, 33.
16. $n = 4$.
17. $x = 40$.
18. child = 6, father = 24.

Review Exercise 5

1. $x = 2$.
2. $x = 15$.
3. $y = \frac{34}{3}$.
4. $x = 2$.
5. $y = \frac{41}{2}$.
6. $y = \frac{13}{2}$.
10. $x = 1$, after checking restrictions.
13. $V = 1386$.
14. $h = 14$.
16. 120 km.
22. $h = 31$.
24. Check denominator restrictions before accepting the solution.