

Chapter 12 | Perimeter and Area of Plane Figures

Secondary 1 Mathematics • Learning and Practice Workbook • Thinkel

Original teaching examples and practice questions modelled on the scope and challenge of the textbook excerpts supplied. All diagrams are schematic, with labelled measurements taking precedence over scale.

Learning objectives

Convert length and area units; find perimeter and area of basic figures; work backwards from area or perimeter; calculate compound and shaded regions; apply the area formulas for parallelograms and trapeziums.

How to use this workbook

Study each worked example, attempt the questions without looking at the answers, and show equations, units and reasoning. Use $\pi = 3.142$ unless otherwise stated.

12.1 Conversion of Units

Length: $1 \text{ m} = 100 \text{ cm}$; $1 \text{ km} = 1000 \text{ m}$. Area: $1 \text{ m}^2 = 10\,000 \text{ cm}^2$; $1 \text{ cm}^2 = 100 \text{ mm}^2$; $1 \text{ km}^2 = 1\,000\,000 \text{ m}^2$. Area conversion factors are squared because both dimensions change.

Worked example 1 | Mixed conversions

- (a) $4.35 \text{ m} = 4.35 \times 100 = 435 \text{ cm}$.
- (b) $0.072 \text{ m}^2 = 0.072 \times 10\,000 = 720 \text{ cm}^2$.
- (c) $18\,500 \text{ cm}^2 = 18\,500 \div 10\,000 = 1.85 \text{ m}^2$.

Practice 12.1

1. Convert (a) 6.08 m to cm , (b) 0.045 km to m , (c) 0.63 m^2 to cm^2 , (d) $42\,000 \text{ mm}^2$ to cm^2 .
2. A rectangular poster measures 1.2 m by 75 cm . Find its area in cm^2 and m^2 .
3. A square courtyard has an area of 0.0144 km^2 . Express its area in m^2 .

12.2 Perimeter and Area of Basic Plane Figures

Perimeter is the total length around a shape. Area is the amount of surface enclosed. Perimeter uses length units (cm, m); area uses square units (cm², m²).

Figure	Perimeter / circumference	Area
Rectangle	$2(l + w)$	lw
Square	$4s$	s^2
Triangle	$a + b + c$	$\frac{1}{2}bh$ (h perpendicular to b)
Circle	$2\pi r = \pi d$	πr^2

Worked example 2 | Rectangle: reverse calculation

A rectangular garden has perimeter 58 m. Its length is 7 m greater than its width. Find its dimensions and area.

Let the width be x m, so length = $(x + 7)$ m.

$$2[x + (x + 7)] = 58 \Rightarrow 4x + 14 = 58 \Rightarrow x = 11.$$

Width = 11 m; length = 18 m.

$$\text{Area} = 18 \times 11 = 198 \text{ m}^2.$$

Worked example 3 | Square

A square has area 169 cm². Its side is $\sqrt{169} = 13$ cm, so its perimeter is $4 \times 13 = 52$ cm.

Practice 12.2A

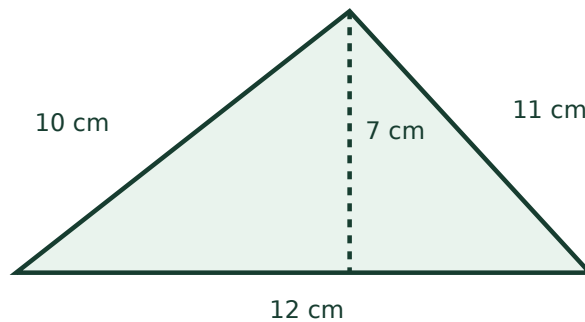
- A rectangle has area 252 cm² and length 21 cm. Find (a) its width, (b) its perimeter.
- A square has perimeter 68 cm. Find its area.
- A rectangular field has perimeter 74 m. Its length is 9 m longer than its width. Find both dimensions and the area.

12.2 Continued | Triangles

For any triangle, perimeter is the sum of its three sides. Area is $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$; the height need not be a side of the triangle.

Worked example 4 | Triangle: two measurements

A triangle has side lengths 10 cm, 11 cm and 12 cm. The perpendicular height to its 12 cm base is 7 cm.



Perimeter = $10 + 11 + 12 = 33$ cm.

Area = $\frac{1}{2} \times 12 \times 7 = 42$ cm².

Worked example 5 | Finding a missing height

A triangle has area 63 cm² and base 18 cm. From $63 = \frac{1}{2} \times 18 \times h$, we get $63 = 9h$, so $h = 7$ cm.

Practice 12.2B

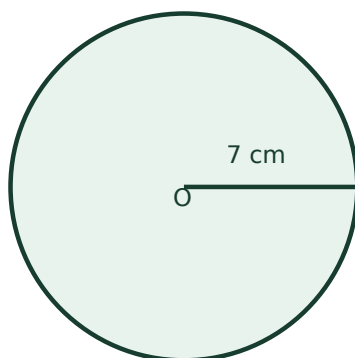
7. A triangle has base 16 cm, perpendicular height 9 cm and other sides 12 cm and 15 cm. Find its perimeter and area.
8. A triangle has area 96 cm² and perpendicular height 12 cm. Find its base.
9. A triangle has sides 13 cm, 14 cm and 15 cm, and a perpendicular height of 12 cm to its 14 cm side. Find (a) its perimeter, (b) its area.

12.2 Continued | Circles

Radius r is the distance from the centre to the circle; diameter $d = 2r$. Circumference is the distance around the circle. For this workbook, use $\pi = 3.142$ unless told otherwise.

Worked example 6 | Circle: circumference and area

A circular pond has radius 7 cm. Find its circumference and area.

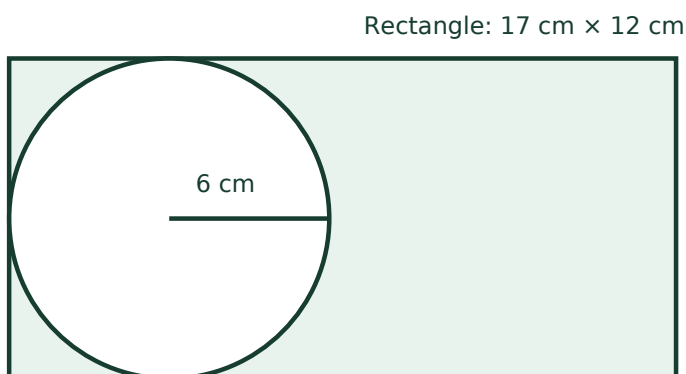


Circumference $= 2 \times 3.142 \times 7 = 43.988 \text{ cm} \approx 44.0 \text{ cm}$ (3 s.f.).

Area $= 3.142 \times 7^2 = 153.958 \text{ cm}^2 \approx 154 \text{ cm}^2$ (3 s.f.).

Worked example 7 | Shaded region

A circle of radius 6 cm fits exactly against the top and bottom edges of a 17 cm $\times$ 12 cm rectangle. Find the area of the region outside the circle but inside the rectangle.



Rectangle area $= 17 \times 12 = 204 \text{ cm}^2$.

Circle area $= 3.142 \times 6^2 = 113.112 \text{ cm}^2$.

Shaded area $= 204 - 113.112 = 90.888 \text{ cm}^2 \approx 90.9 \text{ cm}^2$ (3 s.f.).

Practice 12.2C

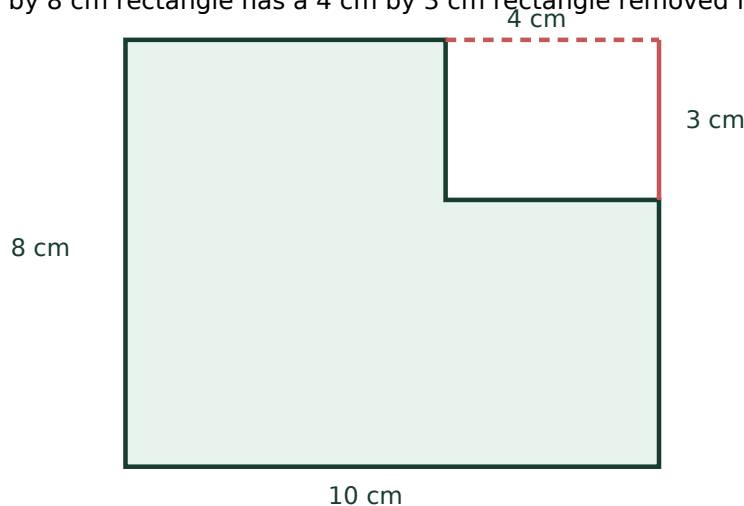
10. Find the circumference and area of a circle with diameter 18 cm. Give answers to 3 s.f.
11. A circle of radius 5 cm is cut from a square of side 12 cm. Find the remaining area to 3 s.f.
12. A semicircle has radius 8 cm. Find its area and perimeter, including the diameter, to 3 s.f.

12.2 Continued | Compound Figures

For compound area, add non-overlapping parts or subtract removed parts. For perimeter, trace only the outside boundary. For curved boundaries, include the relevant fraction of the circumference.

Worked example 8 | Rectangle with a corner removed

A 10 cm by 8 cm rectangle has a 4 cm by 3 cm rectangle removed from its top-right corner.



Original area = $10 \times 8 = 80 \text{ cm}^2$.

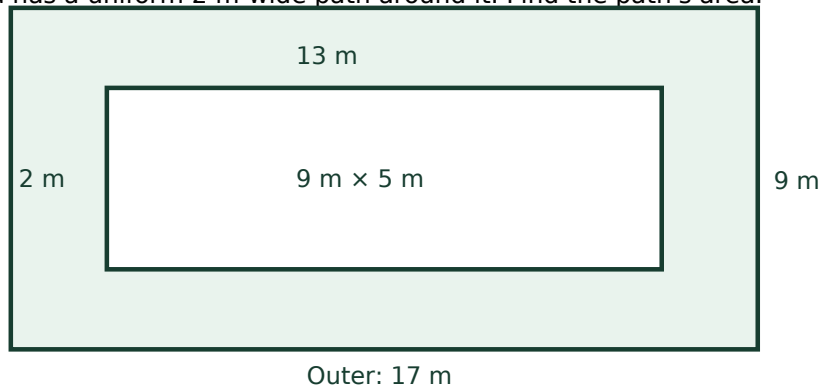
Removed area = $4 \times 3 = 12 \text{ cm}^2$.

Remaining area = $80 - 12 = 68 \text{ cm}^2$.

Outside perimeter = $2(10 + 8) = 36 \text{ cm}$ (the two removed outer segments are replaced by equal-length inner segments).

Worked example 9 | Path around a field

A 13 m $\times$ 5 m field has a uniform 2 m-wide path around it. Find the path's area.



Outer dimensions = $(13 + 2 + 2) \text{ m}$ by $(5 + 2 + 2) \text{ m} = 17 \text{ m} \times 9 \text{ m}$.

Path area = $17 \times 9 - 13 \times 5 = 153 - 65 = 88 \text{ m}^2$.

Practice 12.2D

13. A 15 cm $\times$ 11 cm rectangle has a 5 cm $\times$ 4 cm corner removed. Find its remaining area and outer perimeter.

14. A rectangular lawn measures 18 m $\times$ 10 m. A path 1.5 m wide surrounds it. Find the area of the path.

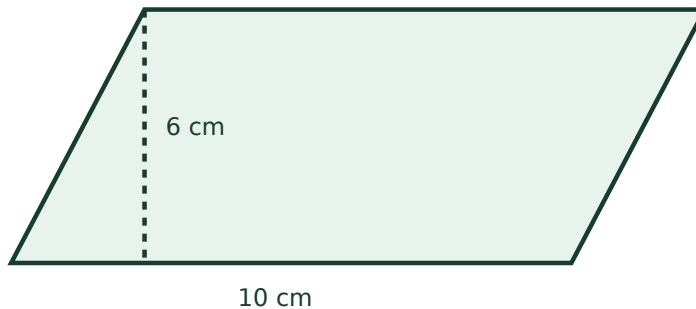
15. A semicircle of radius 4 cm is attached externally along the 8 cm side of an 8 cm $\times$ 10 cm rectangle. Find the total area and outside perimeter to 3 s.f.

12.3 Parallelograms

Area of a parallelogram = base $\times$ perpendicular height. The sloping side is not the height unless it is perpendicular to the base. Perimeter = $2(\text{sum of adjacent side lengths})$.

Worked example 10 | Area and missing height

A parallelogram has base 10 cm and perpendicular height 6 cm. Find its area.



Area = $10 \times 6 = 60 \text{ cm}^2$.

If another parallelogram has area 108 cm^2 and base 12 cm, then $108 = 12h$, so $h = 9 \text{ cm}$.

Worked example 11 | Comparing two bases

The same parallelogram has area 84 cm^2 . One base is 14 cm, and its adjacent side is 12 cm. Find the perpendicular heights corresponding to these two bases.

Height to the 14 cm base = $84 \div 14 = 6 \text{ cm}$.

Height to the 12 cm base = $84 \div 12 = 7 \text{ cm}$.

Practice 12.3

16. Find the area of a parallelogram with base 17 cm and perpendicular height 8 cm.

17. A parallelogram has area 154 cm^2 and base 14 cm. Find its perpendicular height.

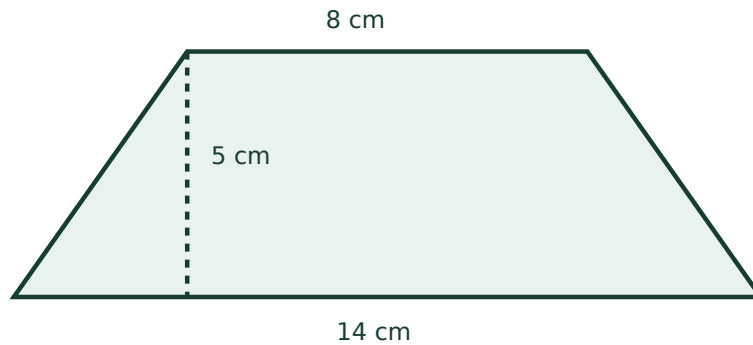
18. A parallelogram has adjacent sides 13 cm and 9 cm. Its perpendicular height to the 13 cm base is 6 cm. Find (a) its area, (b) its perimeter, (c) its height to the 9 cm base.

12.4 Trapeziums

A trapezium has one pair of parallel sides. Area = $\frac{1}{2} \times (\text{sum of the lengths of the parallel sides}) \times \text{perpendicular height}$.

Worked example 12 | Finding the area

A trapezium has parallel sides 8 cm and 14 cm, and perpendicular height 5 cm.



$$\text{Area} = \frac{1}{2} \times (8 + 14) \times 5 = \frac{1}{2} \times 22 \times 5 = 55 \text{ cm}^2.$$

Worked example 13 | Finding an unknown parallel side

A trapezium has area 96 cm^2 , perpendicular height 8 cm, and one parallel side 10 cm. Find the other parallel side x .

$$96 = \frac{1}{2} \times (10 + x) \times 8$$

$$96 = 4(10 + x) \Rightarrow 24 = 10 + x \Rightarrow x = 14 \text{ cm}.$$

Practice 12.4

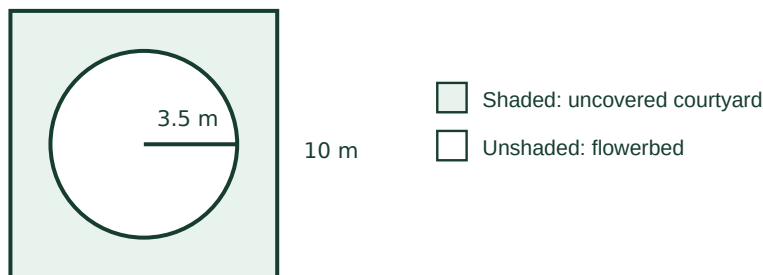
19. Find the area of a trapezium with parallel sides 11 cm and 19 cm and height 7 cm.
20. A trapezium has area 150 cm^2 , height 10 cm and one parallel side 12 cm. Find the other parallel side.
21. A trapezium has parallel sides 9 cm and 15 cm and area 108 cm^2 . Find its perpendicular height.

Mixed Challenge | Textbook-Level Applications

Show clear working. Draw and label a rough sketch where helpful.

22. A rectangular playground has a perimeter of 90 m. Its length is 5 m more than twice its width. Find its dimensions and area.

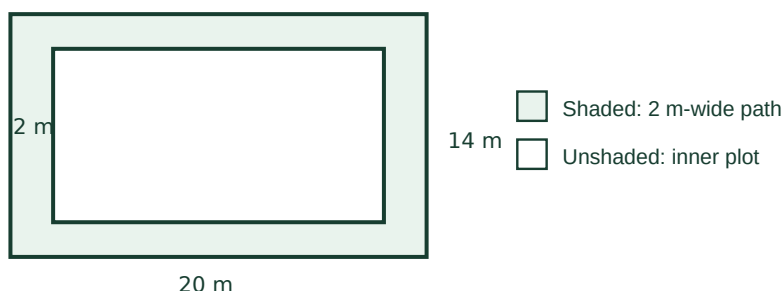
23. A circular flowerbed of radius 3.5 m is built inside a square courtyard of side 10 m. Find the uncovered courtyard area to 3 s.f.



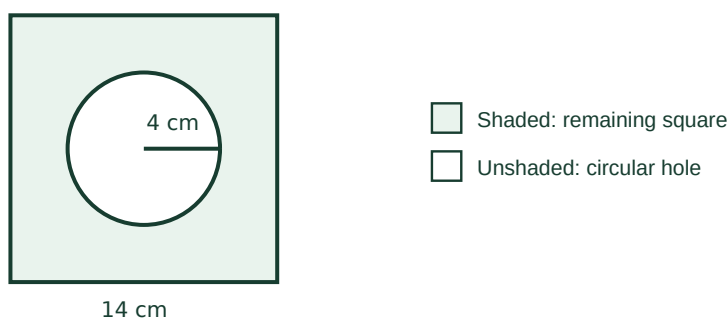
24. A trapezium has area 126 cm^2 and parallel sides in the ratio 3:4. Its perpendicular height is 6 cm. Find the lengths of its parallel sides.

25. A parallelogram and a triangle have equal areas. The parallelogram has base 15 cm and height 8 cm. The triangle has base 20 cm. Find the triangle's perpendicular height.

26. A rectangular plot measures $20 \text{ m} \times 14 \text{ m}$. A 2 m-wide path is built inside the plot along all four sides. Find the area of the path.



27. A square of side 14 cm has a circular hole of radius 4 cm cut from it. Find (a) the remaining area, (b) the total boundary length of the remaining shape, including the circular hole. Give answers to 3 s.f.



Answers and Worked Solutions | 12.1-12.2B

Conversion of units

1. (a) 608 cm; (b) 45 m; (c) 6300 cm²; (d) 420 cm².
2. 1.2 m = 120 cm. Area = $120 \times 75 = 9000 \text{ cm}^2 = 0.9 \text{ m}^2$.
3. $0.0144 \times 1\,000\,000 = 14\,400 \text{ m}^2$.

Rectangles and squares

4. Width = $252 \div 21 = 12 \text{ cm}$. Perimeter = $2(21 + 12) = 66 \text{ cm}$.
5. Side = $68 \div 4 = 17 \text{ cm}$. Area = $17^2 = 289 \text{ cm}^2$.
6. Let width = x ; length = $x + 9$. Then $2(2x + 9) = 74 \Rightarrow 4x = 56 \Rightarrow x = 14$. Dimensions: 14 m and 23 m. Area = $14 \times 23 = 322 \text{ m}^2$.

Triangles

7. Perimeter = $16 + 12 + 15 = 43 \text{ cm}$. Area = $\frac{1}{2} \times 16 \times 9 = 72 \text{ cm}^2$.
8. $96 = \frac{1}{2} \times b \times 12 \Rightarrow 96 = 6b \Rightarrow b = 16 \text{ cm}$.
9. Perimeter = $13 + 14 + 15 = 42 \text{ cm}$. Area = $\frac{1}{2} \times 14 \times 12 = 84 \text{ cm}^2$.

Answers and Worked Solutions | 12.2C-12.2D

Circles

10. Radius = 9 cm. Circumference = $2 \times 3.142 \times 9 = 56.556 \approx 56.6$ cm. Area = $3.142 \times 9^2 = 254.502 \approx 255$ cm².

11. Remaining area = $12^2 - 3.142 \times 5^2 = 144 - 78.55 = 65.5$ cm².

12. Area = $\frac{1}{2} \times 3.142 \times 8^2 = 100.544 \approx 101$ cm². Perimeter = $\pi r + 2r = 3.142 \times 8 + 16 = 41.136 \approx 41.1$ cm.

Compound figures

13. Area = $15 \times 11 - 5 \times 4 = 145$ cm². Outer perimeter = $2(15 + 11) = 52$ cm.

14. Outer dimensions = 21 m $\times$ 13 m. Path area = $21 \times 13 - 18 \times 10 = 273 - 180 = 93$ m².

15. Total area = $8 \times 10 + \frac{1}{2} \times 3.142 \times 4^2 = 80 + 25.136 = 105$ cm² (3 s.f.). Outside perimeter = $10 + 10 + 8 + (3.142 \times 4) = 40.568 \approx 40.6$ cm.

Answers and Worked Solutions | 12.3-12.4

Parallelograms

16. $\text{Area} = 17 \times 8 = 136 \text{ cm}^2$.

17. $\text{Height} = 154 \div 14 = 11 \text{ cm}$.

18. (a) $\text{Area} = 13 \times 6 = 78 \text{ cm}^2$. (b) $\text{Perimeter} = 2(13 + 9) = 44 \text{ cm}$. (c) $\text{Height} = 78 \div 9 = 8\frac{2}{3} \text{ cm}$.

Trapeziums

19. $\text{Area} = \frac{1}{2} \times (11 + 19) \times 7 = 105 \text{ cm}^2$.

20. $150 = \frac{1}{2} \times (12 + x) \times 10 \Rightarrow 30 = 12 + x \Rightarrow x = 18 \text{ cm}$.

21. $108 = \frac{1}{2} \times (9 + 15) \times h \Rightarrow 108 = 12h \Rightarrow h = 9 \text{ cm}$.

Answers and Worked Solutions | Mixed Challenge

22. Let width = x and length = $2x + 5$. Then $2(x + 2x + 5) = 90 \Rightarrow 6x + 10 = 90 \Rightarrow x = 40/3$ m. Length = $95/3$ m. Area = $(40/3)(95/3) = 3800/9 \text{ m}^2 \approx 422 \text{ m}^2$ (3 s.f.).

23. Square area = 100 m^2 . Circle area = $3.142 \times 3.5^2 = 38.4895 \text{ m}^2$. Uncovered area = 61.5 m^2 (3 s.f.).

24. Let the parallel sides be $3x$ and $4x$. Then $126 = \frac{1}{2}(7x)(6) = 21x \Rightarrow x = 6$. Parallel sides = 18 cm and 24 cm.

25. Parallelogram area = $15 \times 8 = 120 \text{ cm}^2$. For triangle, $120 = \frac{1}{2} \times 20 \times h \Rightarrow h = 12 \text{ cm}$.

26. Inner rectangle = $(20 - 4) \times (14 - 4) = 16 \times 10 = 160 \text{ m}^2$. Path area = $20 \times 14 - 160 = 120 \text{ m}^2$.

27. Remaining area = $14^2 - 3.142 \times 4^2 = 196 - 50.272 = 146 \text{ cm}^2$ (3 s.f.). Total boundary = square perimeter + hole circumference = $56 + 2 \times 3.142 \times 4 = 81.136 \approx 81.1 \text{ cm}$ (3 s.f.).

End of Chapter 12 Workbook • Keep all steps and units visible in your own solutions.