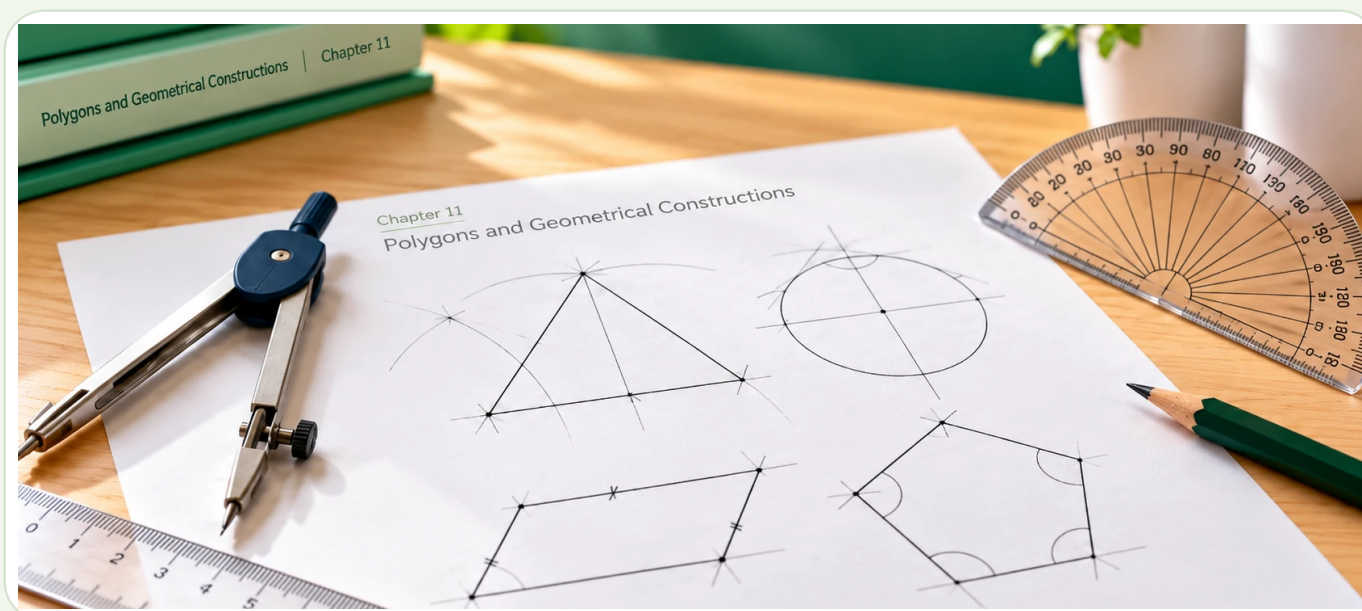


CHAPTER 11

Polygons and Geometrical Constructions

Learning & Practice Workbook



WHAT YOU WILL LEARN

11.1 Triangles
Properties, angles and reasoning.

11.2 Quadrilaterals
Special shapes and diagonals.

11.3 Geometrical constructions
Accurate compass-and-ruler work.

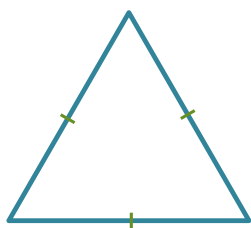
11.4 Polygons
Angle sums and symmetry.

Original teaching explanations, illustrated worked examples, guided practice and answers. Use together with the Chapter 11 classroom page.

11.1 TRIANGLES

Types of triangles

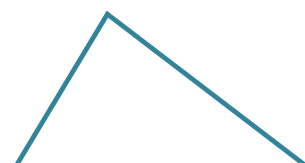
A triangle has three straight sides, three vertices and three interior angles. In $\angle ABC$, the middle letter B is the vertex.

CLASSIFY BY SIDE LENGTH**Equilateral**

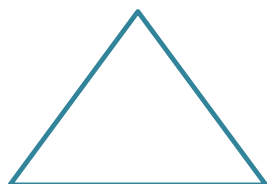
3 equal sides. Each angle is 60° .

Isosceles

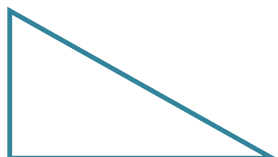
At least 2 equal sides. Base angles are equal.

Scalene

No equal sides. All angles are different.

CLASSIFY BY ANGLES**Acute-angled**

All angles are less than 90° .

Right-angled

One angle is exactly 90° .

Obtuse-angled

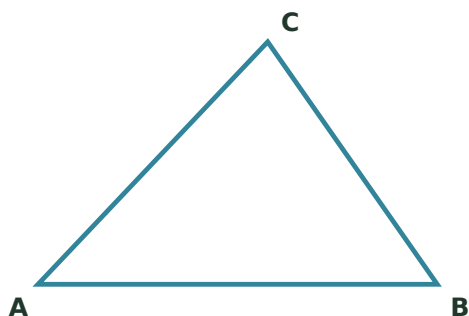
One angle is between 90° and 180° .

Important relationship: An equilateral triangle is also isosceles under the textbook definition (at least two equal sides). A triangle can be classified by both sides and angles.

11.1 TRIANGLES

Side properties and triangle inequality

PROPERTY 1 • Compare sides and opposite angles



Largest angle $\leftrightarrow$ longest opposite side.
Smallest angle $\leftrightarrow$ shortest opposite side.
Equal angles $\leftrightarrow$ equal opposite sides.

A side opposite an angle does not touch the angle's vertex.

WORKED EXAMPLE 1 • Find the longest side

In $\triangle ABC$, $\angle A = 42^\circ$, $\angle B = 79^\circ$ and $\angle C = 59^\circ$. Which side is longest?

Reasoning: $\angle B$ is the largest angle (79°). The side opposite $\angle B$ is AC.

Answer: AC is the longest side.

PROPERTY 2 • Triangle inequality

For three lengths to form a triangle, the sum of any two sides must be greater than the third side. The quickest test is:

shortest side + middle side > longest side

CAN form a triangle

4 cm, 6 cm, 9 cm

$$4 + 6 = 10 > 9$$

Yes. The triangle inequality holds.

CANNOT form a triangle

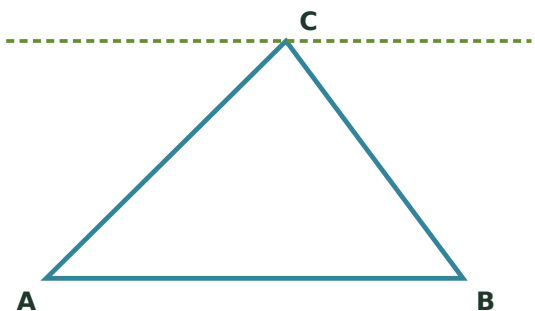
3 cm, 5 cm, 8 cm

$$3 + 5 = 8 \text{ (not greater)}$$

No. The segments lie in a straight line.

11.1 TRIANGLES

Angle sum and exterior angles

ANGLE SUM • Three interior angles add to 180° 

$$\angle A + \angle B + \angle C = 180^\circ.$$

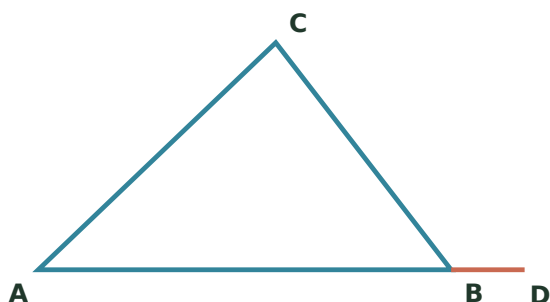
Draw a line through the top vertex parallel to the base. Alternate angles show how all three angles combine to form a straight angle.

WORKED EXAMPLE 2

In $\triangle PQR$, $\angle P = 39^\circ$ and $\angle Q = 74^\circ$. Find $\angle R$.

$\angle R = 180^\circ - 39^\circ - 74^\circ = 67^\circ$ (angle sum of triangle).

EXTERIOR ANGLE • Extending a side



Extend **AB beyond B** to a point D. The exterior angle at B and the adjacent interior angle add to 180° .

Exterior angle = sum of the two interior opposite angles.

WORKED EXAMPLE 3

In $\triangle ABC$, AB is extended beyond B to D. Given $\angle A = 52^\circ$ and $\angle C = 68^\circ$, find exterior $\angle CBD$.

$\angle CBD = 52^\circ + 68^\circ = 120^\circ$ (exterior angle of triangle).

Check: interior $\angle B = 60^\circ$, so $180^\circ - 60^\circ = 120^\circ$.

11.1 PRACTICE

Triangles • guided questions

1 State the type by side lengths: (a) 7, 7, 7 cm; (b) 8, 8, 5 cm; (c) 4, 6, 9 cm.

2 A triangle has angles 90° , 35° and 55° . Classify it by angles.

3 In $\triangle ABC$, $\angle A = 41^\circ$, $\angle B = 78^\circ$ and $\angle C = 61^\circ$. Which side is longest?

4 Can lengths 4 cm, 7 cm and 12 cm form a triangle? Show the inequality.

5 In $\triangle PQR$, $\angle P = 53^\circ$ and $\angle Q = 46^\circ$. Calculate $\angle R$.

6 An isosceles triangle has vertex angle 38° . Find each base angle.

7 In $\triangle XYZ$, side YZ is extended beyond Z to W . If $\angle X = 47^\circ$ and $\angle Y = 66^\circ$, find exterior $\angle XZW$.

11.2 QUADRILATERALS

Angle sum and special quadrilaterals

Quadrilateral angle sum: 360° . One diagonal divides a quadrilateral into two triangles, each with an angle sum of 180° .

SIX IMPORTANT SHAPES

Parallelogram

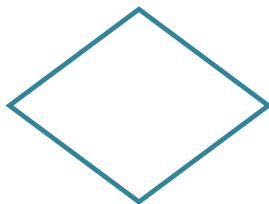
Both pairs of opposite sides parallel.

Rectangle

A parallelogram with 4 right angles.

Square

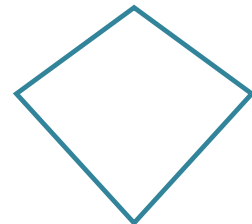
4 equal sides and 4 right angles.

Rhombus

4 equal sides; opposite sides parallel.

Trapezium

At least 1 pair of parallel sides.

Kite

At least 2 pairs of equal adjacent sides.

Textbook definitions matter. The definition of trapezium is inclusive: every parallelogram is also a trapezium. The definition of kite is inclusive: a rhombus is also a kite. A square is both a rectangle and a rhombus.

WORKED EXAMPLE 4

Three interior angles of a quadrilateral are 86° , 104° and 92° .

Fourth angle = $360^\circ - 86^\circ - 104^\circ - 92^\circ = 78^\circ$.

11.2 QUADRILATERALS

Sides, angles and diagonals

PROPERTY GUIDE • Use the correct diagonal rule

Parallelogram

Diagonals bisect each other; opposite angles are equal.

Rectangle

Diagonals bisect each other and are equal.

Rhombus

Diagonals bisect each other at right angles; each diagonal bisects two vertex angles.

Square

Diagonals are equal, perpendicular and bisect the vertex angles.

Kite

Diagonals are perpendicular. The symmetry diagonal bisects the other diagonal.

Trapezium

A general trapezium has no special diagonal equality rule.

WORKED EXAMPLE 5

ABCD is a parallelogram. $\angle DAB = 72^\circ$. Find $\angle ABC$ and $\angle BCD$.

$\angle ABC = 180^\circ - 72^\circ = 108^\circ$ (adjacent angles).

$\angle BCD = 72^\circ$ (opposite angles).

WORKED EXAMPLE 6

ABCD is a rectangle. Diagonals AC and BD meet at E. If $AE = 4$ cm, find BD and ED.

AC = 8 cm (bisected).

BD = 8 cm (equal diagonals).

ED = 4 cm (bisected).

11.2 PRACTICE

Quadrilaterals • guided questions

8

Three angles in a quadrilateral are 105° , 87° and 91° . Find the remaining angle.

9

In parallelogram ABCD, $\angle A = 64^\circ$. Find $\angle B$, $\angle C$ and $\angle D$.

10

Rectangle ABCD has diagonals AC and BD intersecting at E. If $CE = 5.5$ cm, find BD and BE.

11

In rhombus ABCD, $\angle ABC = 116^\circ$. Find $\angle BCD$ and $\angle ABD$ (BD bisects $\angle ABC$).

12

Trapezium ABCD has $AB \parallel CD$. If $\angle DAB = 77^\circ$ and $\angle ABC = 104^\circ$, calculate $\angle ADC$ and $\angle BCD$.

13

Kite ABCD has $AB = AD$, $CB = CD$, and diagonals AC and BD meet at E. If $\angle BAE = 31^\circ$, find $\angle DAE$.

14

Is every square a rectangle? Is every rectangle a square? Explain using properties.

11.3 GEOMETRICAL CONSTRUCTIONS

Instruments and accurate drawings

A construction uses given lengths and angles, not just visual estimation. Keep pencil lines thin, mark intersections carefully and leave compass arcs visible.

RULER

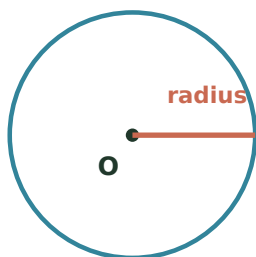
Draw a straight segment and measure its length.

PROTRACTOR

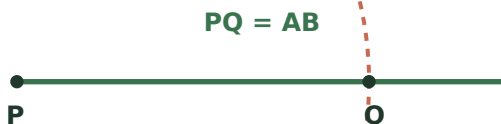
Construct a given angle at the correct vertex.

PAIR OF COMPASSES

Draw an arc or transfer an exact distance.

SKILL 1 • Draw a circle of radius 3 cm

1. Set the compasses to the required radius using a ruler.
2. Place the pointed end at centre O.
3. Rotate the drawing end without changing the opening.

SKILL 2 • Transfer a length using compasses

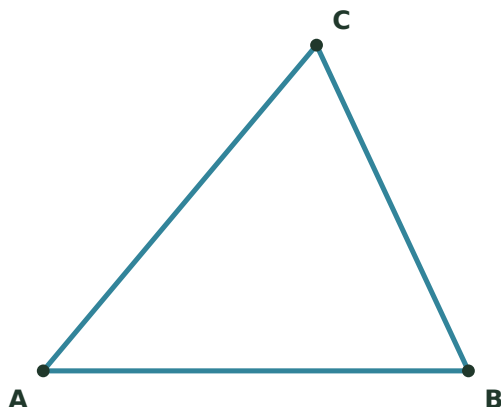
Construction tip: Mark where the arcs cross. Use a small dot or cross, then join the required vertices. Printed diagrams are illustrative; use the stated measurements for your own construction.

11.3 GEOMETRICAL CONSTRUCTIONS

Constructing triangles • two methods

METHOD A • One side and two angles

Construct $\triangle ABC$: $AB = 7\text{ cm}$, $\angle A = 50^\circ$, $\angle B = 65^\circ$.

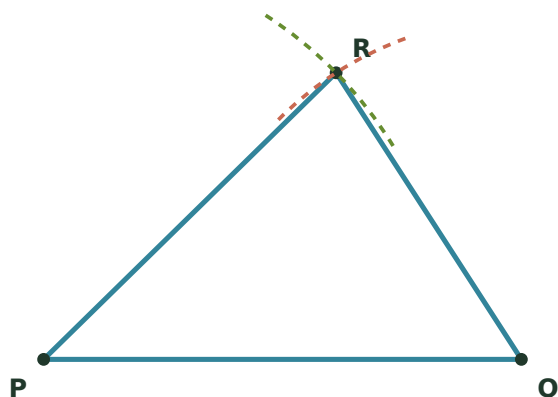
**Steps**

1. Draw $AB = 7\text{ cm}$.
2. At A, draw a ray making 50° with AB.
3. At B, draw a ray making 65° with BA, on the same side.
4. Label the intersection C.

Check: $\angle C = 180^\circ - 50^\circ - 65^\circ = 65^\circ$.

METHOD B • Three sides (SSS)

Construct $\triangle PQR$: $PQ = 7\text{ cm}$, $PR = 6\text{ cm}$, $QR = 5\text{ cm}$.

**Steps**

1. Draw $PQ = 7\text{ cm}$.
2. With P as centre, draw an arc of radius 6 cm.
3. With Q as centre, draw an arc of radius 5 cm.
4. Mark their intersection R and join PR, QR.

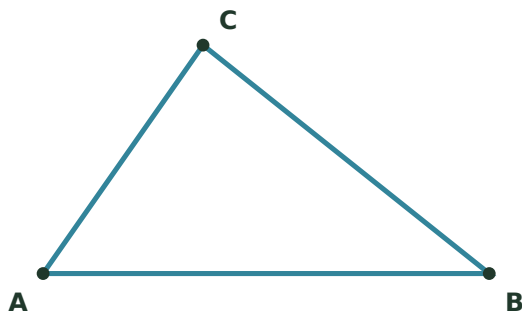
Check: $5 + 6 > 7$, so this triangle can exist.

11.3 GEOMETRICAL CONSTRUCTIONS

Two sides and an angle • when shapes differ

METHOD C • Included angle (SAS)

Construct $\triangle ABC$: $AB = 8\text{ cm}$, $AC = 5\text{ cm}$, $\angle BAC = 55^\circ$.



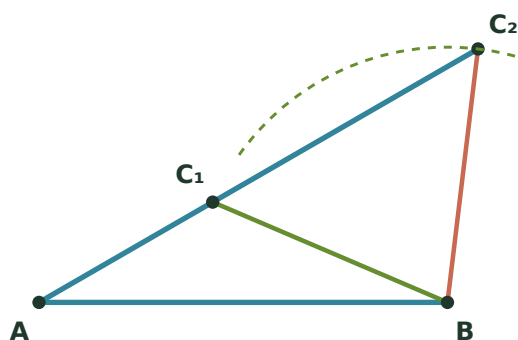
Steps

1. Draw $AB = 8\text{ cm}$.
2. Construct the 55° angle at A.
3. Mark C on the angle ray, 5 cm from A.
4. Join BC.

The two known sides meet at the known angle; the triangle is determined up to reflection.

SPECIAL CASE • A non-included angle may give two triangles

Given $AB = 8\text{ cm}$, $BC = 5\text{ cm}$, $\angle BAC = 30^\circ$.



Steps

1. Draw $AB = 8\text{ cm}$.
2. Draw a 30° ray from A.
3. Draw an arc centred at B with radius 5 cm .
4. It cuts the ray at C_1 and C_2 .

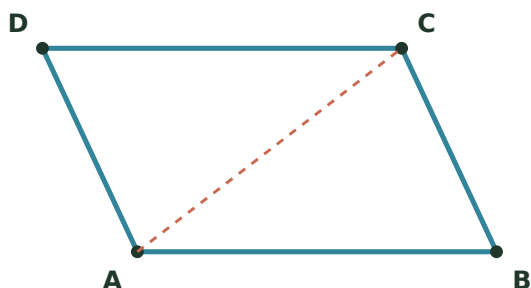
Conclusion: Both triangles meet the measurements but have different shapes.

11.3 GEOMETRICAL CONSTRUCTIONS

Constructing quadrilaterals

CONSTRUCT A PARALLELOGRAM

$AB = 8 \text{ cm}$, $BC = 5 \text{ cm}$ and $\angle ABC = 65^\circ$.

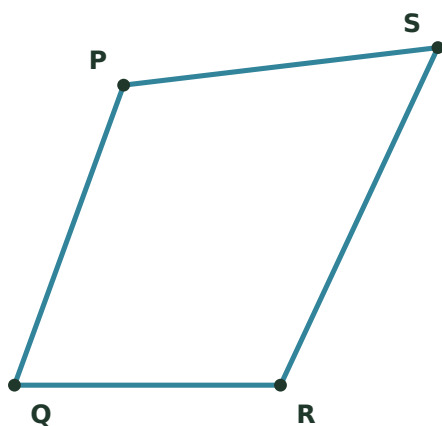


Construction steps

1. Draw $AB = 8 \text{ cm}$.
2. At B, construct $\angle ABC = 65^\circ$.
3. Mark $BC = 5 \text{ cm}$ on that ray.
4. With A as centre, draw an arc of radius 5 cm.
5. With C as centre, draw an arc of radius 8 cm.
6. Their intersection on the opposite side of AC from B is D; join AD and CD.
7. Check opposite sides are equal and parallel.

CONSTRUCT A GENERAL QUADRILATERAL

$PQ = 6 \text{ cm}$, $QR = 5 \text{ cm}$, $RS = 7 \text{ cm}$, $\angle PQR = 70^\circ$, $\angle QRS = 115^\circ$.



Construction steps

1. Draw $QR = 5 \text{ cm}$.
2. At Q, construct the 70° ray; mark P at 6 cm from Q.
3. At R, construct the 115° ray from RQ on the same side as P.
4. Mark S at 7 cm from R.
5. Join PS and check all measurements.

Four side lengths alone do not always fix a general quadrilateral.

11.3 PRACTICE

Constructions • use real instruments

Practical work: Use a sharp pencil, ruler, protractor and pair of compasses. Construct each figure on separate paper at the stated lengths. Keep construction arcs and label all vertices.

- 15** Construct $\triangle ABC$ with $AB = 7$ cm, $\angle BAC = 48^\circ$ and $\angle ABC = 67^\circ$. Calculate $\angle ACB$.

- 16** Construct $\triangle PQR$ with $PQ = 6$ cm, $PR = 5$ cm and $QR = 4$ cm. Explain how the compass arcs locate R.

- 17** Construct $\triangle DEF$ with $DE = 8$ cm, $DF = 5$ cm and $\angle EDF = 45^\circ$ (included angle).

- 18** Explain why lengths 3 cm, 4 cm and 8 cm cannot form a triangle.

- 19** Construct a parallelogram ABCD with $AB = 7$ cm, $BC = 4$ cm and $\angle ABC = 65^\circ$. State two checks.

- 20** Given $AB = 8$ cm, $BC = 5$ cm and $\angle BAC = 30^\circ$, show how the arc centred at B gives two possible locations for C.

11.4 POLYGONS

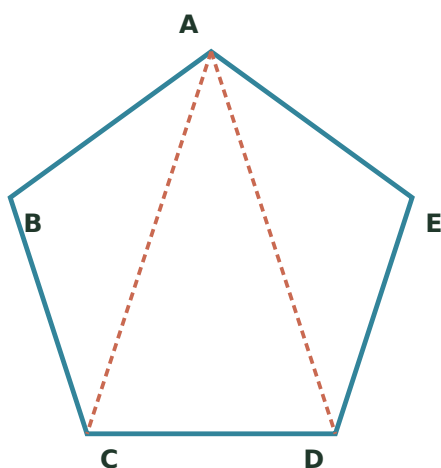
Recognise polygons and find angle sums

WHAT COUNTS AS A POLYGON?

A polygon is a **closed** plane figure made from straight line segments. In this section we study **simple convex polygons**: the sides do not cross and every interior angle is less than 180° .

A regular polygon has **all sides equal** and **all interior angles equal**. Equal sides alone are not enough.

INTERIOR ANGLE SUM • Divide the figure into triangles



From a single vertex of a convex n -sided polygon, draw diagonals to all non-adjacent vertices.

Number of triangles = $n - 2$

Sum of interior angles = $(n - 2) \times 180^\circ$

For a pentagon: $(5 - 2) \times 180^\circ = \mathbf{540^\circ}$.

WORKED EXAMPLE 7 • A regular decagon

A regular decagon has 10 sides. Find its interior angle sum and each interior angle.

Sum = $(10 - 2) \times 180^\circ = 1440^\circ$.

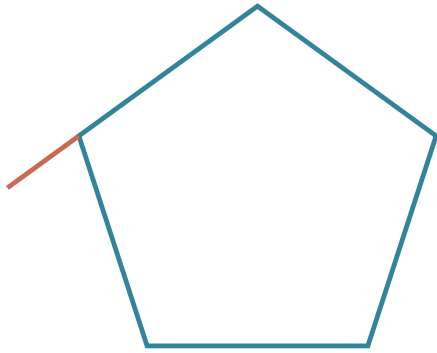
Each interior angle = $1440^\circ \div 10 = 144^\circ$.

Remember: A concave polygon is not studied in detail here. Do not confuse a simple convex polygon with a star made from crossing line segments.

11.4 POLYGONS

Exterior angles and regular polygons

EXTERIOR ANGLE SUM • One full turn



At each vertex, extend a side and mark the exterior angle in the same direction around the polygon.

Sum of one exterior angle at each vertex = 360° .

These exterior turns make one complete rotation.

FORMULAS FOR A REGULAR n -SIDED POLYGON

Each exterior angle = $360^\circ \div n$

Each interior angle = $180^\circ - 360^\circ \div n$

Number of sides $n = 360^\circ \div \text{exterior angle}$

WORKED EXAMPLE 8

A regular polygon has exterior angle 24° . How many sides?

$$n = 360^\circ \div 24^\circ = 15 \text{ sides.}$$

WORKED EXAMPLE 9

Find each interior angle of a regular hexagon ($n = 6$).

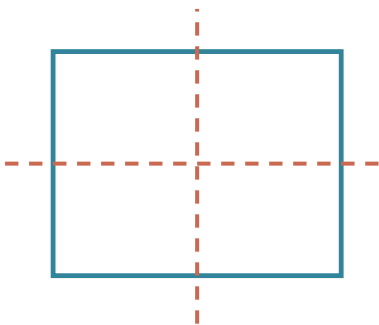
$$\text{Exterior} = 360^\circ \div 6 = 60^\circ.$$

$$\text{Interior} = 180^\circ - 60^\circ = 120^\circ.$$

11.4 POLYGONS

Line symmetry and rotational symmetry

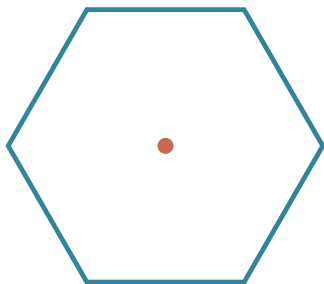
LINE SYMMETRY • A mirror fold



A **line of symmetry** divides a shape into two matching halves. Folding along this line would make the halves overlap exactly.

A non-square rectangle has **2 lines of symmetry**: one horizontal and one vertical through its centre.

ROTATIONAL SYMMETRY • Turning about the centre



The **order of rotational symmetry** is the number of times a figure fits exactly onto itself in one complete turn (360°).

A regular hexagon has order **6**, because it matches after every 60° turn. Every figure also matches at 360° .

QUICK COMPARISON • Common figures

Shape	Symmetry lines	Rotational order
Non-square rectangle	2	2
Square	4	4
Non-square rhombus	2	2
Regular pentagon	5	5
Regular hexagon	6	6

11.4 PRACTICE

Polygons • guided questions

21 Explain why a closed shape made of five straight, non-crossing sides is a polygon.

22 Find the sum of the interior angles of an octagon (8 sides).

23 Find each interior angle of a regular hexagon. State the formula used.

24 Each exterior angle of a regular polygon is 30° . Find the number of sides.

25 The interior angles of a pentagon are 100° , 110° , 125° , 95° and x° . Find x .

26 Each interior angle of a regular polygon is 156° . Find its number of sides.

27 How many lines of symmetry and what rotational order does a non-square rectangle have?

28 A regular pentagon has how many lines of symmetry? State its rotational order.

CHAPTER 11 REVIEW

Mixed reasoning challenge

Use a diagram where helpful. Give a reason for each angle calculation. For construction problems, use the measurements given; sketches in this workbook are not for direct measurement.

- 29** In $\triangle ABC$, BC extends beyond C to D. Exterior $\angle ACD = 132^\circ$ and $\angle A = 56^\circ$. Find $\angle B$ and interior $\angle C$.

- 30** In rhombus ABCD, $\angle ABC = 110^\circ$. Diagonal BD bisects $\angle ABC$. Find $\angle ABD$ and $\angle BCD$.

- 31** A regular 12-sided polygon: find (a) the interior angle sum, (b) each interior angle, (c) each exterior angle.

- 32** Construct $\triangle ABC$ given $AB = 8$ cm, $BC = 5$ cm and $\angle BAC = 30^\circ$. Explain the two possible positions for C.

- 33** The exterior angles of a convex quadrilateral are 72° , 95° , 113° and x° . Find x .

- 34** Trapezium ABCD has $AB \parallel CD$. $\angle DAB = 73^\circ$ and $\angle ABC = 118^\circ$. Find $\angle ADC$ and $\angle BCD$.

ANSWERS • 11.1

Triangles • worked answers

1

(a) Equilateral (b) Isosceles (c) Scalene

Compare the number of equal side lengths.

2

Right-angled triangleOne angle measures exactly 90° .

3

AC is the longest side $\angle B = 78^\circ$ is the largest angle; AC is opposite $\angle B$.

4

No triangle is possible $4 + 7 = 11$, which is less than 12. Triangle inequality fails.

5

 $\angle R = 81^\circ$ $180^\circ - 53^\circ - 46^\circ = 81^\circ$ (angle sum of triangle).

6

Each base angle = 71° $(180^\circ - 38^\circ) \div 2 = 71^\circ$ (equal base angles).

7

 $\angle XZW = 113^\circ$ $47^\circ + 66^\circ = 113^\circ$ (exterior angle of triangle).

Quadrilaterals • worked answers

8

 77°

$$360^\circ - 105^\circ - 87^\circ - 91^\circ = 77^\circ.$$

9

 $\angle B = 116^\circ$, $\angle C = 64^\circ$, $\angle D = 116^\circ$ Adjacent angles sum to 180° ; opposite angles are equal.

10

 $BD = 11$ cm; $BE = 5.5$ cm $AC = 2 \times CE = 11$ cm. Rectangle diagonals are equal and bisect one another.

11

 $\angle BCD = 64^\circ$; $\angle ABD = 58^\circ$ $180^\circ - 116^\circ = 64^\circ$; $116^\circ \div 2 = 58^\circ$ (rhombus diagonal bisects $\angle B$).

12

 $\angle ADC = 103^\circ$; $\angle BCD = 76^\circ$ $180^\circ - 77^\circ = 103^\circ$; $180^\circ - 104^\circ = 76^\circ$ (interior angles, $AB \parallel CD$).

13

 $\angle DAE = 31^\circ$ AC is the symmetry diagonal of the kite and bisects $\angle BAD$.

14

Yes; no

Every square has four right angles, so it is a rectangle. A rectangle may have unequal adjacent sides.

ANSWERS • 11.3

Constructions • methods and checks

15

 $\angle ACB = 65^\circ$ $180^\circ - 48^\circ - 67^\circ = 65^\circ$. Draw AB, then both angle rays and mark their intersection.

16

Use two intersecting compass arcs

With P as centre, radius 5 cm; with Q as centre, radius 4 cm. Intersection gives R.

17

One included-angle constructionDraw $DE = 8$ cm; construct 45° at D; mark F at $DF = 5$ cm; join EF.

18

Cannot be constructed $3 + 4 = 7$, less than 8; the triangle inequality is not satisfied.

19

Check opposite sides are equal and parallel $AB = CD = 7$ cm; $BC = AD = 4$ cm; $AB \parallel CD$ and $BC \parallel AD$.

20

There are two possible positions for CThe 5 cm arc centred at B intersects the 30° ray from A at two positive points. Both satisfy the givens.

ANSWERS • 11.4

Polygons • worked answers

21

It is a (simple) pentagon

It is closed, uses straight non-crossing sides and has five sides.

22

1080°

$$(8 - 2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ.$$

23

120°Each exterior angle = $360^\circ \div 6 = 60^\circ$. Each interior = $180^\circ - 60^\circ = 120^\circ$.

24

12 sides

$$n = 360^\circ \div 30^\circ = 12.$$

25

x = 110°

$$\text{Pentagon sum} = 540^\circ. x = 540^\circ - (100^\circ + 110^\circ + 125^\circ + 95^\circ) = 110^\circ.$$

26

15 sides

$$\text{Each exterior angle} = 180^\circ - 156^\circ = 24^\circ; n = 360^\circ \div 24^\circ = 15.$$

27

2 symmetry lines; rotational order 2A non-square rectangle has horizontal and vertical mirror axes and matches after a 180° turn.

28

5 symmetry lines; rotational order 5Every regular pentagon has five mirror axes and matches after each 72° turn.

Mixed reasoning • worked answers

29

$$\angle B = 76^\circ; \angle C = 48^\circ$$

$$\angle B = 132^\circ - 56^\circ = 76^\circ \text{ (exterior angle). } \angle C = 180^\circ - 132^\circ = 48^\circ \text{ (straight line).}$$

30

$$\angle ABD = 55^\circ; \angle BCD = 70^\circ$$

$$\angle ABD = 110^\circ \div 2 = 55^\circ \text{ (diagonal bisects angle). } \angle BCD = 180^\circ - 110^\circ = 70^\circ.$$

31

$$\text{(a) } 1800^\circ \text{ (b) } 150^\circ \text{ (c) } 30^\circ$$

$$\text{Sum} = (12 - 2) \times 180^\circ = 1800^\circ. \text{ Interior} = 1800^\circ \div 12 = 150^\circ. \text{ Exterior} = 360^\circ \div 12 = 30^\circ.$$

32

Two different triangles are possible

Draw the 30° ray at A; an arc centred at B of radius 5 cm cuts it at two positions for C.

33

$$x = 80^\circ$$

$$\text{Exterior angles sum to } 360^\circ. x = 360^\circ - (72^\circ + 95^\circ + 113^\circ) = 80^\circ.$$

34

$$\angle ADC = 107^\circ; \angle BCD = 62^\circ$$

$$180^\circ - 73^\circ = 107^\circ; 180^\circ - 118^\circ = 62^\circ \text{ (interior angles, } AB \parallel CD \text{).}$$

KEY FORMULAS

Triangle: 180° • Quadrilateral: 360° • Polygon interior sum: $(n - 2) \times 180^\circ$

Exterior sum: 360° • Regular polygon exterior angle: $360^\circ \div n$